Calibration of the Deterministic and Stochastic Volatility Libor Market Model

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- Deterministic Volatility Libor Market Model
    - Calibration to Cap Volatilities
    - Calibration to Swaption Volatilities
- Stochastic Volatility Libor Market Model
    - Calibration to the Cap Smile

1. BGM/Jamshidian-Model (Deterministic Volatility Libor Market Model)

The log-normal Libor Market Model under the terminal bond numeraire Q^{N+1}:

$$\mathrm{d}\,L_i(t) = L_i(t)\left(\mu_i^{Q^{N+1}}(t)\mathrm{d}\,t + \mathrm{d}\,W_i(t)\right) \tag{1}$$

with

$$\mu_i^{Q^{N+1}}(t) = -\sum_{j=i+1}^{N} \frac{\delta_j L_j(t)\,\mathrm{cov}_{ij}(t)}{1+\delta_j L_j(t)} \quad , \tag{2}$$

$$\langle \mathrm{d}\,W_i(t)\mathrm{d}\,W_j(t)\rangle = \mathrm{cov}_{ij}(t)\mathrm{d}\,t \quad , \tag{3}$$

where given

- tenor structure $0 = T_0 < T_1 < T_2 < \cdots < T_{N+1}$, $\delta_i = T_{i+1} - T_i$
- Libor rates $L_i(t) = \frac{1}{\delta_i}\left(\frac{B(t,T_i)}{B(t,T_{i+1})} - 1\right)$
- zero coupon bond prices $B(t,T)$ with maturity T

Decomposition to a s-factor model ($s \leq N$):

$$\mathrm{d}\,L_i(t) = L_i(t)\left(-\sum_{j=i+1}^{N}\frac{\delta_j L_j(t)\,\mathrm{cov}_{ij}(t)}{1+\delta_j L_j(t)}\mathrm{d}\,t + \sum_{p=0}^{s}\gamma_{ip}(t)\mathrm{d}\,Z_p(t)\right) \quad , \tag{4}$$

where

- γ_{ip} are the instantaneous loadings for the factor $Z_p(t)$
- $Z_p(t)$ uncorrelated one dimensional Wiener processes
- $\mathrm{cov}_{ij}(t) = \sum_{p=0}^{s}\gamma_{ip}(t)\gamma_{jp}(t)$

Connection to the forward swap rates:

$$SR_{ij}(t) = \frac{B(t,T_i) - B(t,T_{j+1})}{\sum_{k=i}^{j}\delta_k B(t,T_{k+1})} = \frac{\sum_{k=i}^{j}\delta_k L_k(t) B(t,T_{k+1})}{\sum_{k=i}^{j}\delta_k B(t,T_{k+1})} \quad . \tag{5}$$

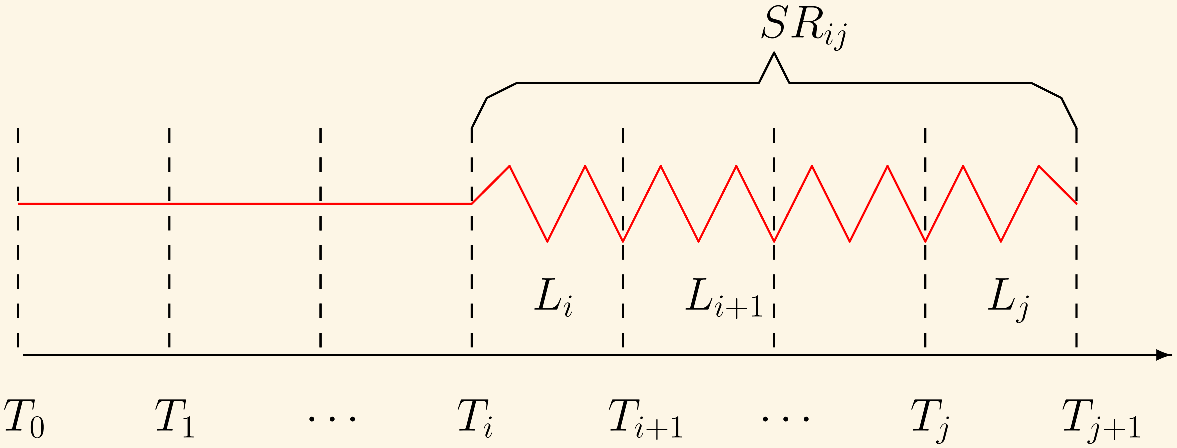


Figure 1: The reset and expiry times for the forward SWAP rate SR_{ij} and embedded forward Libor rates $L_{k=i,\ldots,j}$.

2. Forward Libor Rate Volatility and Calibration to Cap Volatilities

Instantaneous forward Libor rate volatility:

$$\sigma_i^{\text{inst}}(t)^2 = \sum_{p=1}^{s} \gamma_{ip}(t)^2 \tag{6}$$

Black caplet volatilities:

$$\left(\sigma_i^{\text{Black}}\right)^2 T_i \quad = \quad \int_0^{T_i} \mathrm{d}\, t \left(\sigma_i^{\text{inst}}(t)\right)^2 \tag{7}$$

However, flat cap volatilities are quoted in the market (all caps have identical implied volatility)! There are infinitely ways of calculated caplet volatilities from cap volatilities(see Rebonato). An realistic time evolution of the term structure of forward rate volatilities can be achieved by assuming time homogeneity

$$\begin{aligned} \gamma_{ip}(t) &= \gamma_p(T_i - t) && (8) \\ \sigma_i^{\text{inst}}(t) &= \sigma^{\text{inst}}(T_i - t) && (9) \\ \text{cov}_{ij}(t) &= \text{cov}(T_i - t, T_j - t) \quad . && (10) \end{aligned}$$

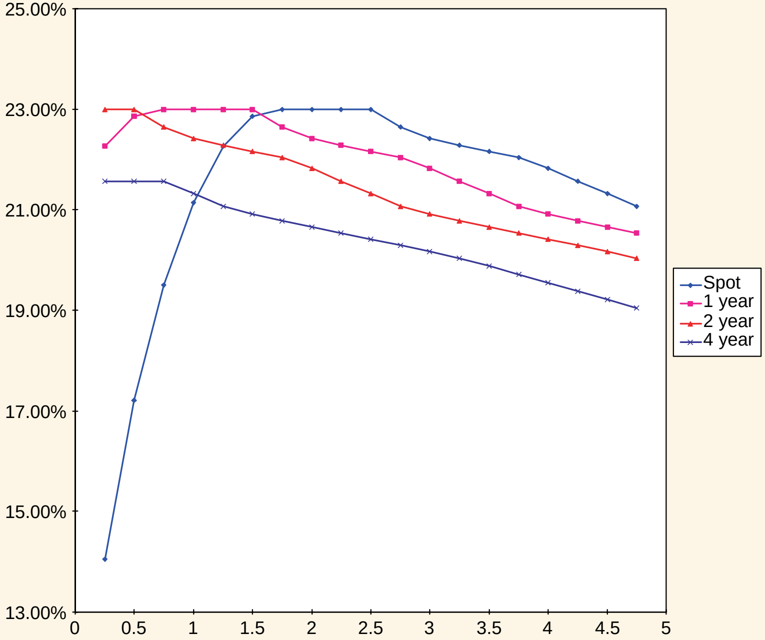


Figure 2: Taken from Rebonato's book: Assuming a time independent volatility functions for the varous forward rates implies as the time evolution of the volatility curve a translation of the spot curve to the left along the maturity axis.

Caplet volatilities using time homogeneous forward rate volatilities:

$$\begin{aligned}\left(\sigma_i^{\text{Black}}\right)^2 T_i &= \int_0^{T_i} \mathrm{d}\,t \left(\sigma^{\text{inst}}(T_i - t)\right)^2 && (11)\\ &= \int_0^{T_i} \mathrm{d}\,t \left(\sigma^{\text{inst}}(t)\right)^2 && (12)\end{aligned}$$

- functional form for a humped volatility curve:

$$\sigma^{\text{inst}}(t) = (a + b\,t)\,\mathrm{e}^{-c\,t} + d \tag{13}$$

- calculate caplet volatilities (integrate $\sigma^{\text{inst}}(t)$; closed form solution)
- cap $= \sum$ caplets
- least square optimization of cap price differences

Perfect calibration to at-the-money caplet volatilities is only possible by introducing a non time homogeneous scaling factor k_i:

$$\sigma_i^{\text{inst}}(t) = k(T_i) * g(T_i - t) \tag{14}$$

where $g(t)$ is a time homogeneous function as above and $k(T_i)$ close to one.

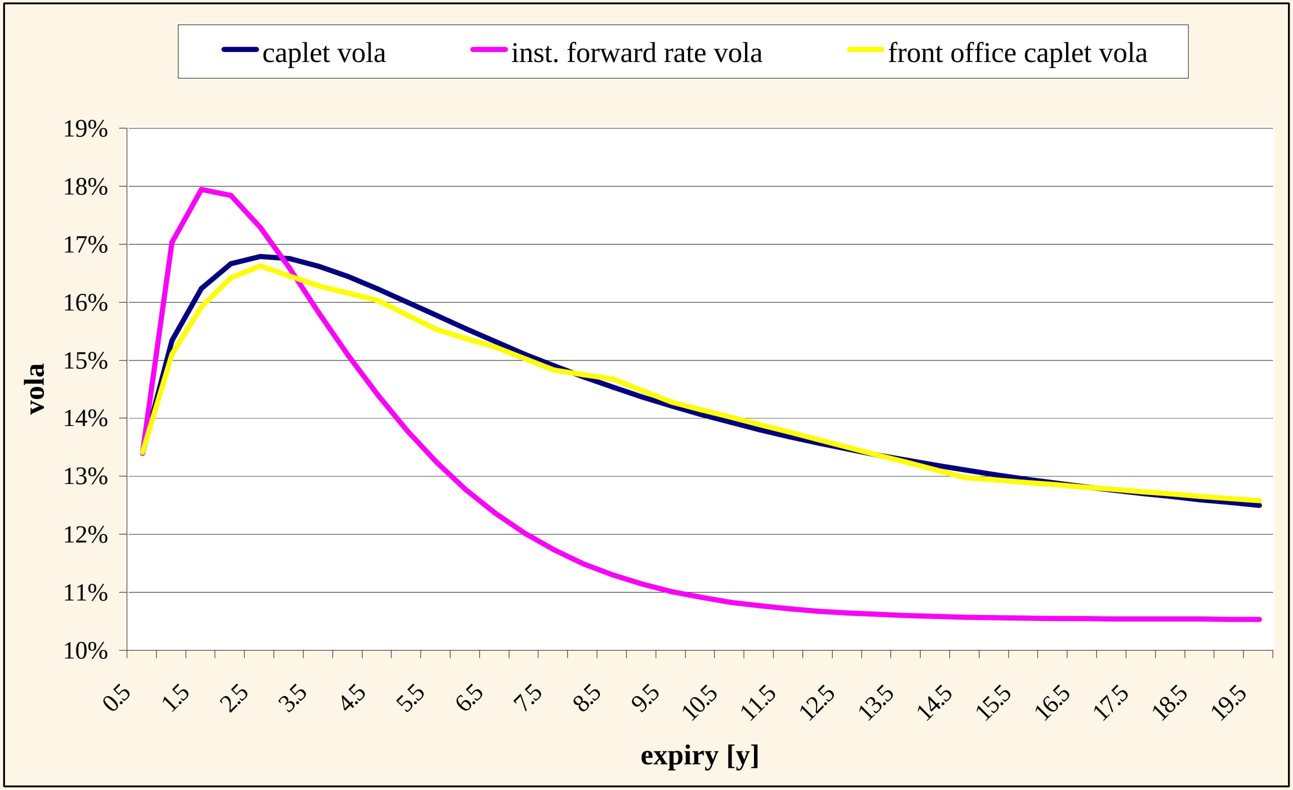


Figure 3: The fitted forward Libor rate volatility (without scaling factor) and resulting spot caplet volatility calibrated to market quotes. For comparison the caplet volatility of the front office (obtained by bootstapping) is shown.

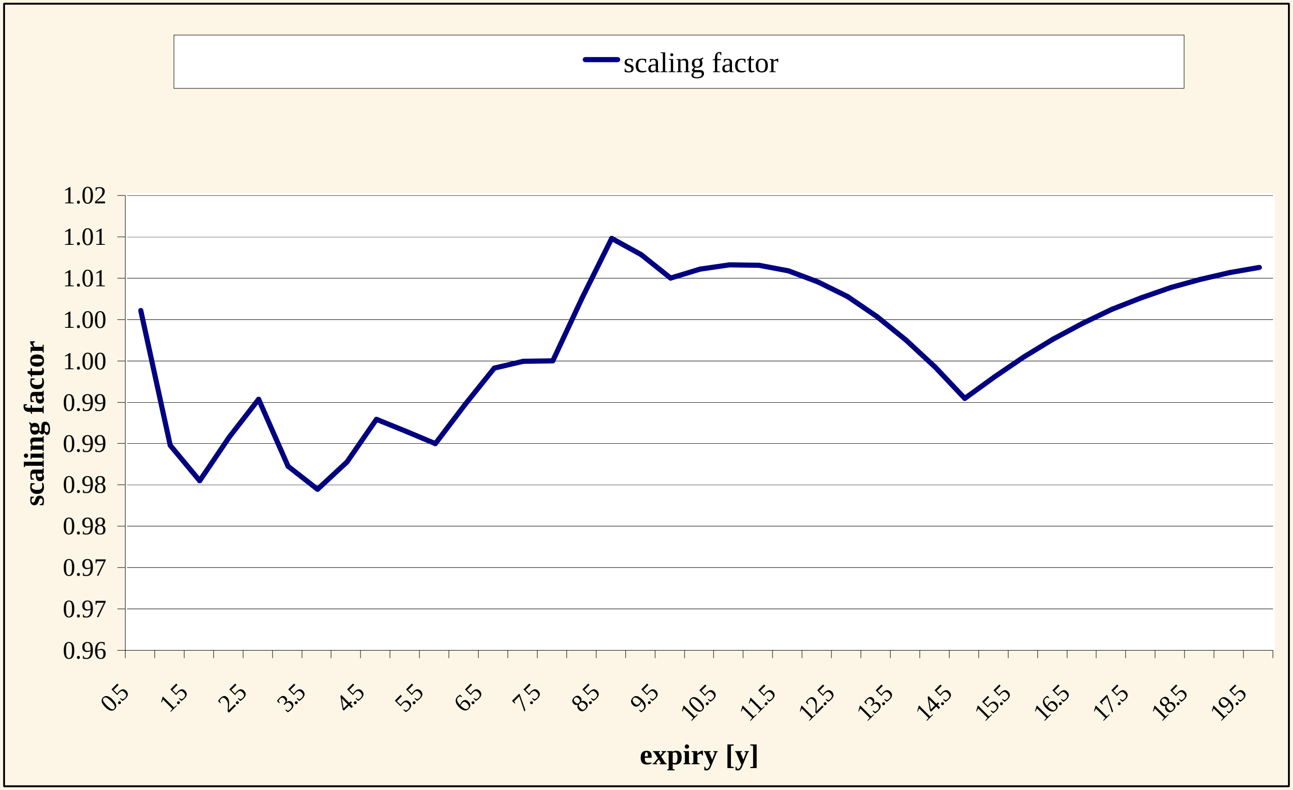


Figure 4: Scaling factor to ensure perfect calibration to caplet volotilies.

3. Forward SWAP Rate Volatility and Calibration to Swaption Volatilities

- forward Libor and forward swap rate can not be simoultaneously lognormal distributed (only true for one factor models)
- but this is a good approximation!

Applying Ito's lemma to the forward swap rates:

$$\left(\sigma_{\mathrm{SR}_{ij}}^{\mathrm{inst}}(t)\mathrm{SR}_{ij}(t)\right)^2 = \sum_{k,l=i}^{j} L_k(t)\frac{\partial \mathrm{SR}_{ij}}{\partial L_k}(t)\,\mathrm{cov}_{kl}(t)\frac{\partial \mathrm{SR}_{ij}}{\partial L_l}(t)L_l(t) \tag{15}$$

approximation:

$$\left(\sigma_{\mathrm{SR}_{ij}}^{\mathrm{inst}}(t)\mathrm{SR}_{ij}(0)\right)^2 \approx \sum_{k,l=i}^{j} L_k(0)\frac{\partial \mathrm{SR}_{ij}}{\partial L_k}(0)\,\mathrm{cov}_{kl}(t)\frac{\partial \mathrm{SR}_{ij}}{\partial L_l}(0)L_l(0) \quad . \tag{16}$$

swaption volatility:

$$\left(\sigma_{\mathrm{SR}_{ij}}^{\mathrm{Black}}\right)^2 T_i = \int_0^{T_i} \mathrm{d}\,t \left(\sigma_{\mathrm{SR}_{ij}}^{\mathrm{inst}}(t)\right)^2 \tag{17}$$

$$\approx \int_0^{T_i} \mathrm{d}\,t \frac{1}{\left(\mathrm{SR}_{ij}(0)\right)^2} \sum_{k,l=i}^{j} L_k(0)\frac{\partial \mathrm{SR}_{ij}}{\partial L_k}(0)\,\mathrm{cov}_{kl}(t)\frac{\partial \mathrm{SR}_{ij}}{\partial L_l}(0)L_l(0) \tag{18}$$

$$= \int_0^{T_i} \mathrm{d}\,t \sum_{k,l=i}^{j} W_{ij}^k \operatorname{cov}_{kl}(t) W_{ij}^l \tag{19}$$

where

$$W_{ij}^k = \frac{L_k(0)}{\mathrm{SR}_{ij}(0)} \frac{\partial \mathrm{SR}_{ij}}{\partial L_k}(0) \tag{20}$$

3.1. Rebonato approximation

- Assume that forward Libor rates are independent of the Bond values :

$$\frac{\partial \mathrm{SR}_{ij}}{\partial L_k}(t) \approx \frac{\delta_k B(t, T_{k+1})}{\sum_{l=i}^{j} \delta_l B(t, T_{l+1})} \tag{21}$$

$$W_{ij}^k = \frac{B(0, T_k) - B(0, T_{k+1})}{B(0, T_i) - B(0, T_{j+1})} \quad . \tag{22}$$

- good approximation for flat yield curves
- fast to compute

3.2. Hull White approximation

- The partial derivatives in equation (15) can be calculated exactly
- result:

$$W_{ij}^{k} \quad = \quad \frac{B(0,T_i)\sum_{m=k}^{j}\delta_m B(0,T_{m+1}) + B(0,T_{j+1})\sum_{m=i}^{k-1}\delta_m B(0,T_{m+1})}{B(0,T_k)\sum_{m=i}^{j}\delta_m B(0,T_{m+1})} \tag{23}$$

- slower to compute
- less approximations are involved
- in case of calibration use first the Rebonato approximation to find good solutions and for the final iterations use the Hull White approximation!

3.3. Calibration of the Implied Correlation

- we have time homogeneous forward rate volatility, need time homogeneous correlation function

$$\rho_{ij}(t) \quad = \quad \rho(T_i - t, T_j - t) = \frac{\mathrm{cov}(T_i - t, T_j - t)}{\sigma^{\mathrm{inst}}(T_i - t)\sigma^{\mathrm{inst}}(T_j - t)} \quad . \tag{24}$$

- for swaption volatilities we need to calculate

$$\begin{aligned} K^i_{kl} &= \int_0^{T_i} \mathrm{d}\, t \, \mathrm{cov}_{kl}(t) && (25)\\ &= \int_0^{T_i} \mathrm{d}\, t \, \mathrm{cov}(T_k - t, T_l - t) && (26)\\ &= \int_0^{T_i} \mathrm{d}\, t \sigma^{\mathrm{inst}}(T_k - t) \rho(T_k - t, T_l - t) \sigma^{\mathrm{inst}}(T_l - t) \quad . && (27) \end{aligned}$$

3.3.1. Functional Form of the Implied Correlation

- The simples functional form:

$$\rho(\tau_1, \tau_2) = \beta_1 + (1 - \beta_1) \exp\left(-\beta_2 |\tau_1 - \tau_2|\right) \tag{28}$$

- this exponential correlation function is the only one, which is not time dependent, i.e.

$$\rho_{ij}(t) = \rho(T_i - T_j) = rho_{ij} \tag{29}$$

- integration can be performed in closed solution, we need only linear algebra to calulate swaption volatilities

- but is unrealistic (long dated rates have the same correlation as short dated rates)

- more realistic:

$$\rho(\tau_1,\tau_2) = \mathrm{LongCorr} + (1-\mathrm{LongCorr})\exp\left(-\beta|\tau_1-\tau_2|\right) \tag{30}$$

with

$$\begin{aligned}\mathrm{LongCorr} &\equiv \beta_1 + \beta_4(\tau_1+\tau_2) + \beta_5|\tau_1-\tau_2| && (31)\\ \beta &\equiv \beta_2 + \beta_3 \max(\tau_1,\tau_2) \quad . && (32)\end{aligned}$$

- integration hase to be done numerically

3.3.2. Functional Ansatz for the Factor Loadings

- direct approach for low dimensional factorization (here onyl two factors):

$$\sigma^{\mathrm{inst}}(\tau)^2 = \gamma_1(\tau)^2 + \gamma_2(\tau)^2 \quad . \tag{33}$$

where

$$\begin{aligned}\gamma_1(\tau) &= \sigma^{\mathrm{inst}}(\tau)\frac{1}{\sqrt{2}}\left(g(\tau) - \sqrt{1-g(\tau)^2}\right) && (34)\\ \gamma_2(\tau) &= \sigma^{\mathrm{inst}}(\tau)\frac{1}{\sqrt{2}}\left(g(\tau) + \sqrt{1-g(\tau)^2}\right) \quad . && (35)\end{aligned}$$

The correlation function becomes

$$\rho(\tau_1,\tau_2) = g(\tau_1)g(\tau_2) + \sqrt{1-g(\tau_1)^2}\sqrt{1-g(\tau_2)^2} \quad . \tag{36}$$

- A good Ansatz is

$$g(\tau) = g_\infty + (1 - g_\infty)\,\mathrm{e}^{-\beta\tau} \quad . \tag{37}$$

3.3.3. Free Form of the Implied Correlation with Smoothing Constrains

- best fitting results with a non parametric form for the correlation:

$$\begin{aligned} \gamma_p(T_i - T_m) &\longrightarrow b_{p,i-m} && (38)\\ \sigma^{\text{inst}}(T_i - T_m) &\longrightarrow s_{i-m} && (39)\\ \rho(T_k - T_m, T_l - T_m) &\longrightarrow \alpha_{k-m,l-m} \quad . && (40)\end{aligned}$$

- approximation of the integrals by sums
  diagonal part:

$$\left(\sigma_i^{\text{Black}}\right)^2 T_i = \int_0^{T_i} \mathrm{d}\,t\,\left(\sigma^{\text{inst}}(T_i - t)\right)^2 = \sum_{m=1}^{i} \delta_{m-1} s_{i-m}^2 = K_{ii}^i \tag{41}$$

  non diagonal part:

$$K_{kl}^i = \int_0^{T_i} \mathrm{d}\,t \sigma^{\text{inst}}(T_k - t)\rho(T_k - t, T_l - t)\sigma^{\text{inst}}(T_l - t) = \sum_{m=1}^{i} \delta_{m-1} s_{k-m} \alpha_{k-m,l-m} s_{l-m} \quad . \tag{42}$$

- time homogeneity implies now $\delta_k = \delta$ for all k

$$s^2_{i-m} = \frac{1}{\delta_{m-1}} \int_{T_i - T_m}^{T_i - T_{m-1}} \mathrm{d}\, t \left(\sigma^{\text{inst}}(t)\right)^2 \quad ; \tag{43}$$

$$s^2_j = \frac{1}{\delta} \int_{T_j}^{T_{j+1}} \mathrm{d}\, t \left(\sigma^{\text{inst}}(t)\right)^2 \quad ; \tag{44}$$

- calibration of correlation matrix $\alpha_{ij} = \sum_{p=1}^{s} b_{pi} b_{pj}$ is now a constrained optimization of the b-loadings
- using spherical coordinates for a unconstrained optimization:

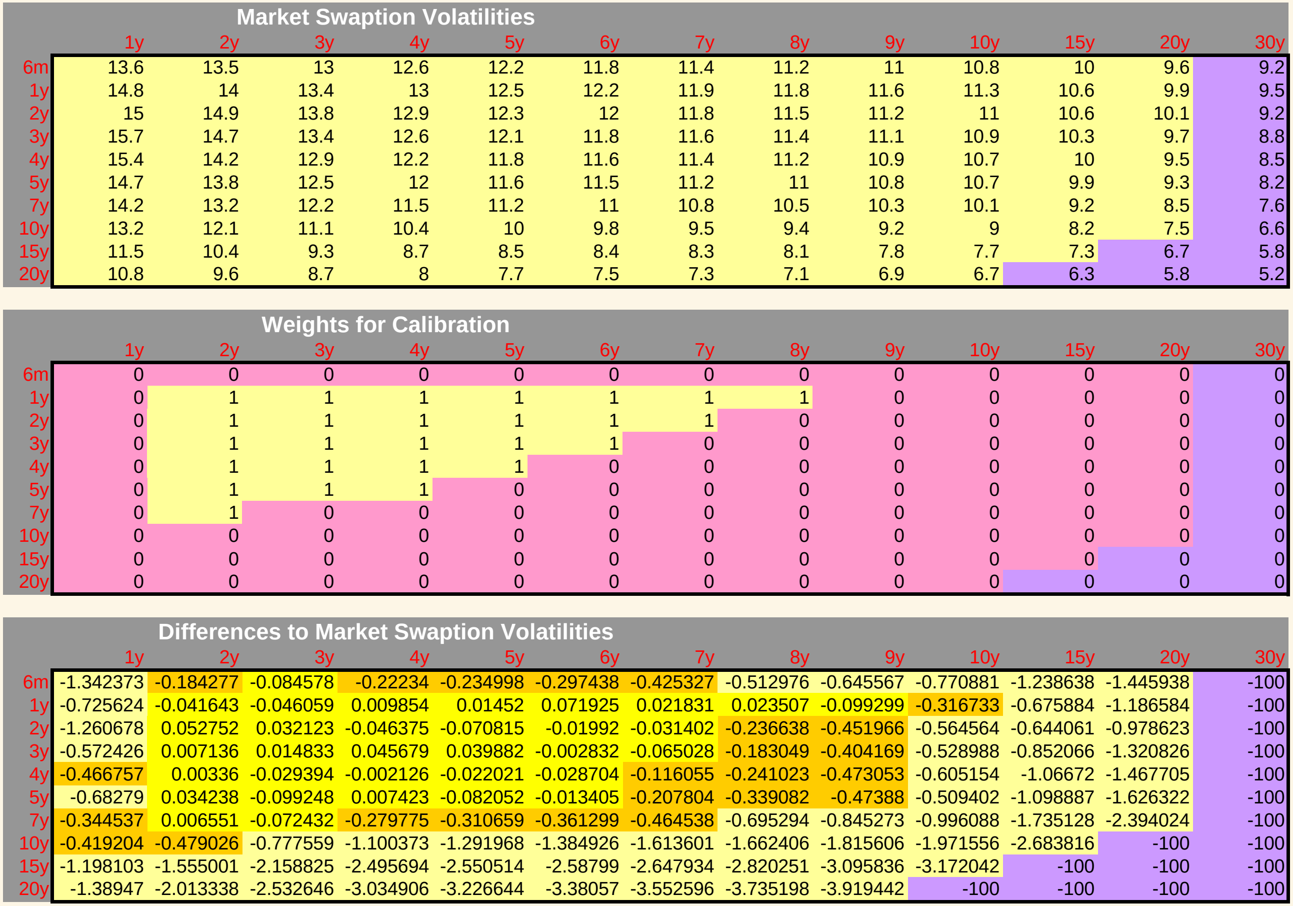
$$b_{pk} = \cos(\theta_{pk}) \prod_{q=1}^{p-1} \sin(\theta_{qk}) \quad ; \quad p = 1, \ldots, s-1 \tag{45}$$

$$b_{sk} = \prod_{q=1}^{s-1} \sin(\theta_{qk}) \quad . \tag{46}$$

- problem:
  - to many free angles θ_{pk} compared to to input parameter
  - noise in the input parameters (bid-ask-spread etc.)

- solution: additional smoothing cost function

$$V^{\text{smoothing}} = \sum_{k=1,l=2}^{i} \left(\alpha_{k,l} - \alpha_{k,l-1}\right)^2 \quad . \tag{47}$$



Market Swaption Volatilities

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	30y
6m	13.6	13.5	13	12.6	12.2	11.8	11.4	11.2	11	10.8	10	9.6	9.2
1y	14.8	14	13.4	13	12.5	12.2	11.9	11.8	11.6	11.3	10.6	9.9	9.5
2y	15	14.9	13.8	12.9	12.3	12	11.8	11.5	11.2	11	10.6	10.1	9.2
3y	15.7	14.7	13.4	12.6	12.1	11.8	11.6	11.4	11.1	10.9	10.3	9.7	8.8
4y	15.4	14.2	12.9	12.2	11.8	11.6	11.4	11.2	10.9	10.7	10	9.5	8.5
5y	14.7	13.8	12.5	12	11.6	11.5	11.2	11	10.8	10.7	9.9	9.3	8.2
7y	14.2	13.2	12.2	11.5	11.2	11	10.8	10.5	10.3	10.1	9.2	8.5	7.6
10y	13.2	12.1	11.1	10.4	10	9.8	9.5	9.4	9.2	9	8.2	7.5	6.6
15y	11.5	10.4	9.3	8.7	8.5	8.4	8.3	8.1	7.8	7.7	7.3	6.7	5.8
20y	10.8	9.6	8.7	8	7.7	7.5	7.3	7.1	6.9	6.7	6.3	5.8	5.2

Weights for Calibration

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	30y
6m	0	0	0	0	0	0	0	0	0	0	0	0	0
1y	0	1	1	1	1	1	1	1	0	0	0	0	0
2y	0	1	1	1	1	1	1	0	0	0	0	0	0
3y	0	1	1	1	1	1	0	0	0	0	0	0	0
4y	0	1	1	1	1	0	0	0	0	0	0	0	0
5y	0	1	1	1	0	0	0	0	0	0	0	0	0
7y	0	1	0	0	0	0	0	0	0	0	0	0	0
10y	0	0	0	0	0	0	0	0	0	0	0	0	0
15y	0	0	0	0	0	0	0	0	0	0	0	0	0
20y	0	0	0	0	0	0	0	0	0	0	0	0	0

Differences to Market Swaption Volatilities

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	30y
6m	-1.342373	-0.184277	-0.084578	-0.22234	-0.234998	-0.297438	-0.425327	-0.512976	-0.645567	-0.770881	-1.238638	-1.445938	-100
1y	-0.725624	-0.041643	-0.046059	0.009854	0.01452	0.071925	0.021831	0.023507	-0.099299	-0.316733	-0.675884	-1.186584	-100
2y	-1.260678	0.052752	0.032123	-0.046375	-0.070815	-0.01992	-0.031402	-0.236638	-0.451966	-0.564564	-0.644061	-0.978623	-100
3y	-0.572426	0.007136	0.014833	0.045679	0.039882	-0.002832	-0.065028	-0.183049	-0.404169	-0.528988	-0.852066	-1.320826	-100
4y	-0.466757	0.00336	-0.029394	-0.002126	-0.022021	-0.028704	-0.116055	-0.241023	-0.473053	-0.605154	-1.06672	-1.467705	-100
5y	-0.68279	0.034238	-0.099248	0.007423	-0.082052	-0.013405	-0.207804	-0.339082	-0.47388	-0.509402	-1.098887	-1.626322	-100
7y	-0.344537	0.006551	-0.072432	-0.279775	-0.310659	-0.361299	-0.464538	-0.695294	-0.845273	-0.996088	-1.735128	-2.394024	-100
10y	-0.419204	-0.479026	-0.777559	-1.100373	-1.291968	-1.384926	-1.613601	-1.662406	-1.815606	-1.971556	-2.683816	-100	-100
15y	-1.198103	-1.555001	-2.158825	-2.495694	-2.550514	-2.58799	-2.647934	-2.820251	-3.095836	-3.172042	-100	-100	-100
20y	-1.38947	-2.013338	-2.532646	-3.034906	-3.226644	-3.38057	-3.552596	-3.735198	-3.919442	-100	-100	-100	-100

Figure 5: A crucial point in the calibration of the correlation matrix is the selection of the reference swaptions.

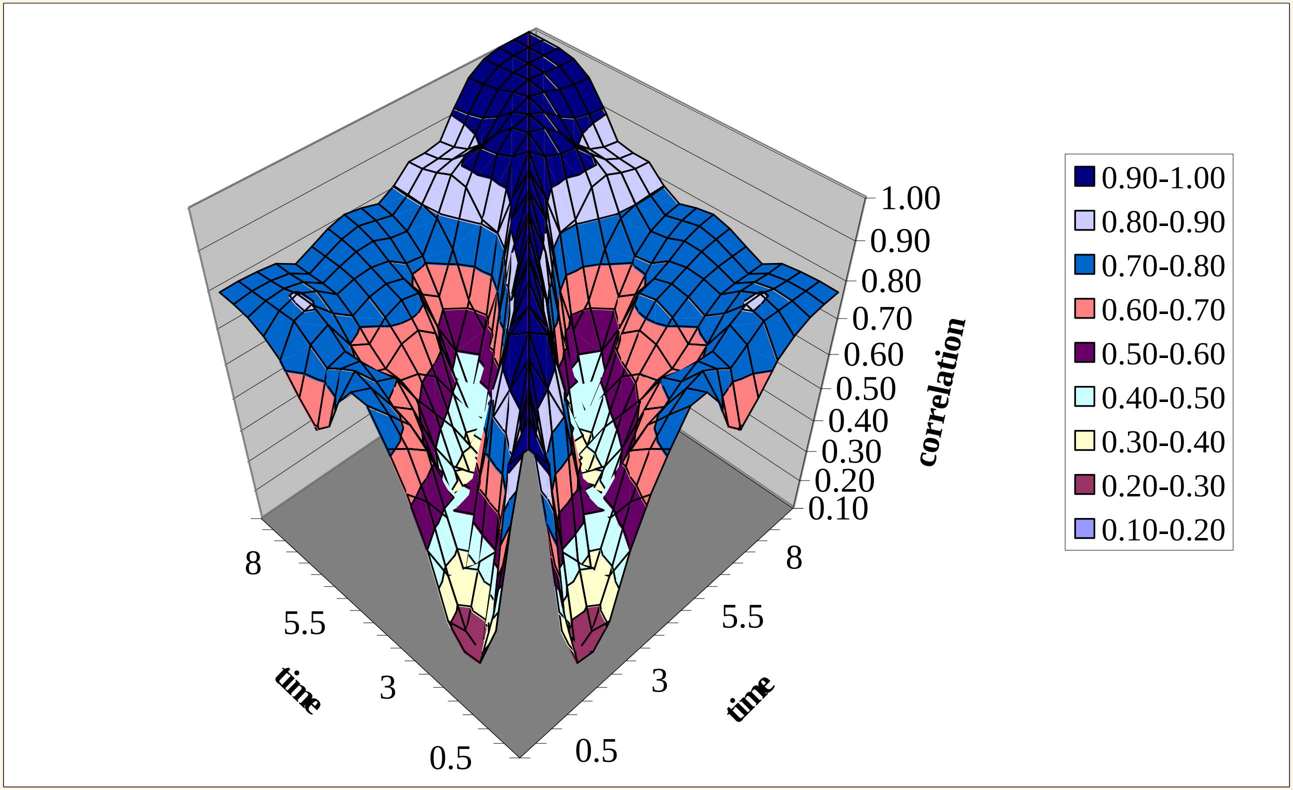


Figure 6: Implied correlation function obtained from a 5 factor model calibrated to cap and swaption volatilities.

4. Stochastic Volatility Extension of the Libor Market Model

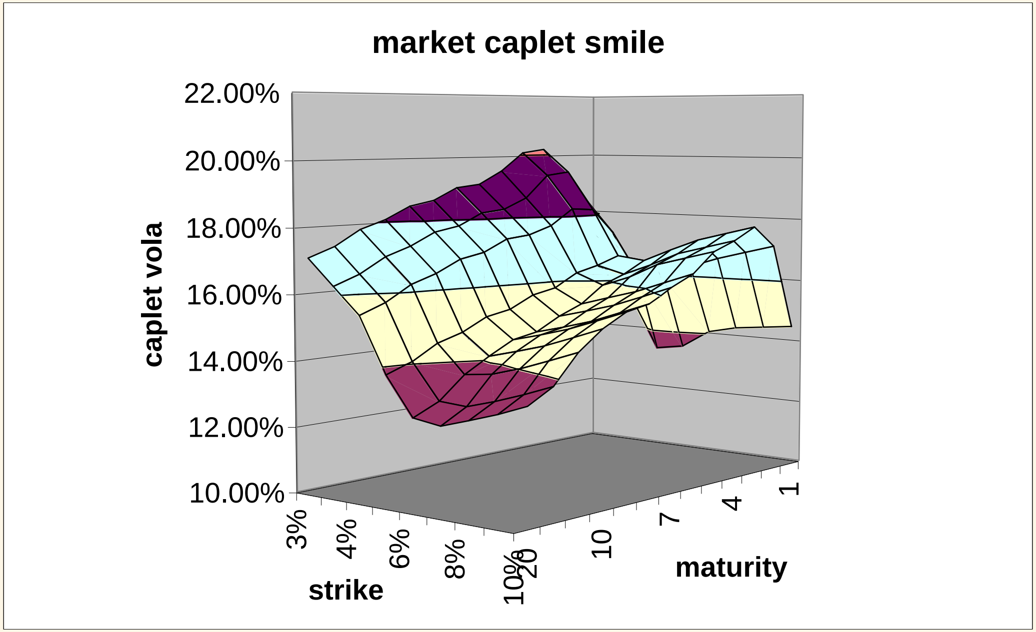


Figure 7: Market caplet volatility smile.

4.1. Smile/Skew modeling by other authors:

- Andersen and Andreasen: Constant Elasticity of Variance (CEV) Model
- Brigo and Mercurio: Mixture of Lognormal Model
- ???: Shifted Lognormal (SL) Model
- Rebonato: Stochastic parameters for the functional form of instantaneous volatility (a, b, c, d)

4.2. Stochastic Volatility Extension

- Heston style stochastic volatility and CEV/SL extension:

$$\begin{aligned} \mathrm{d}\, L_i(t) &= \phi(L_i(t)) \left(\mu_i^{Q^{N+1}}(t) \mathrm{d}\, t + g_i(t) \sqrt{V_i(t)} \mathrm{d}\, W_i(t) \right) && (48) \\ \mathrm{d}\, V_i(t) &= \lambda_i(t) \left(\bar{V}_i(t) - V_i(t) \right) \mathrm{d}\, t + \eta_i(t) \sqrt{V_i(t)} \mathrm{d}\, Z_i(t) \quad , && (49) \end{aligned}$$

where,

$\phi(L)$: function to model CEV or SL
$g_i(t)$: deterministic part of volatility
$\lambda_i(t)$: mean reversion speed
$\bar{V}_i(t)$: long term mean of V
$\eta_i(t)$: volatility of variance

- CEV model for $\phi(L) = L^{\gamma}$ $(0 < \gamma < 2)$
- SL model for $\phi(L) = L + \alpha$.
- closed form solution for both models is known
- we will model the skew with the $\phi(L)$, smile with the Heston stochastic volatility (i.e. Z is uncorrelated to W)
    - this **assumption** allows a technical simplification
    - empirically justified that caplet volatilities are uncorrelated to Libor rates.
- technical simplification:
    - can adapt the Hull & White (1987)

$$C_i = \int DS(x_i)\Phi(x_i)\mathrm{d}\,x_i \tag{50}$$

where C_i is the price of the i-th caplet,
$\cdots$ $DS(x)$ is the solutions for deterministic volatility,
$\cdots$ $\Phi(x)$ is the probability density of the realized variance which can be computed by Monte Carlo simulation of

$$x_i = \frac{1}{T_i}\int_0^{T_i} g_i(t)^2 V_i(t)\mathrm{d}\,t \tag{51}$$

- this Hull & White result can also applied to the european swaption approximation
- in the following we will use the Shifted Lognormal Model only
  (the solution of the deterministic volatility problem is a modified Black76 solution)

4.3. Euler Simulation Schema for Stochastic Volatility Libor Market Model

- using Ito's lemma:

$$\begin{aligned} \mathrm{d}\,\ln(L_i(t)+\alpha) &= \left(\mu_i^{Q^{N+1}}(t) - \frac{1}{2} g_i(t)^2 V_i(t)\right) \mathrm{d}\,t + g_i(t)\sqrt{V_i(t)}\mathrm{d}\,W_i(t) && (52) \\ \mathrm{d}\,2\sqrt{V_i(t)} &= \frac{1}{\sqrt{V_i(t)}}\left(\lambda_i(t)\left(\bar{V}_i(t) - V_i(t)\right) - \frac{1}{4}\eta^2\right) \mathrm{d}\,t + \eta_i(t)\mathrm{d}\,Z_i(t) \quad . && (53) \end{aligned}$$

- Euler difference schema:

$$\begin{aligned} L_i(t_{k+1}) &= (L_i(t_k)+\alpha)\exp\left\{\left(\mu_i^{Q^{N+1}}(t_k) - \frac{1}{2} g_i(t_k)^2 V_i(t_k)\right)\Delta + g_i(t_k)\sqrt{V_i(t_k)}\sqrt{\Delta}W_i(k)\right\} - \alpha \\ V_i(t_{k+1}) &= \left(\sqrt{V_i(t_k)} + \frac{1}{2\sqrt{V_i(t_k)}}\left(\lambda_i(t_k)\left(\bar{V}_i(t_k) - V_i(t_k)\right) - \frac{1}{4}\eta_i(t_k)^2\right)\Delta + \eta_i(t)\sqrt{\Delta}Z_i(k)\right)^2 \end{aligned}$$

  with $\Delta = t_{k+1} - t_k$ and $W_i(k), Z_i(k)$ drawn from Gaussian distribution $N(1,0)$

- note: need to simulate the stochastic volatility part only

4.4. Time Homogeneous Parameterization

- as for the deterministic volatility Libor market model all model parameters should be time homogenous, i.e.

$$\begin{aligned} g_i(t) &= g(T_i - t) && (54)\\ \lambda_i(t) &= \lambda(T_i - t) && (55)\\ \bar{V}_i(t) &= \bar{V}(T_i - t) && (56)\\ \eta_i(t) &= \eta(T_i - t) && (57)\\ \langle \mathrm{d}\, Z_i(t) \mathrm{d}\, Z_j(t) \rangle &= \rho(T_i - t, T_j - t) \mathrm{d}\, t \quad . && (58) \end{aligned}$$

- "perfect" calibration to at-the-money or fixed strike caplet volatilities can be obtained by utilization of the initial conditions of the hidden stochastic volatility process, i.e.

$$V_i(0) \neq \bar{V}_i(0) = \bar{V}(T_i) \tag{59}$$

4.5. Simples Implementation

- smallest model setup =
  - one stochastic volatility factor model
  - use a functional form for the humped forward rate volatility curve.

$$g(t) \quad = \quad (a + b\, t)\, \mathrm{e}^{-c\, t} + d \tag{60}$$

  - reduce the number of model parameter to the minimum number

$$\begin{aligned} \lambda(t) &= \lambda \quad (61) \\ \bar{V}(t) &= 1 \quad (62) \\ \eta(t) &= \eta \quad (63) \\ \rho(t,t') &= 1 \quad (64) \end{aligned}$$

- note 1: we have still one stochastic volatility equation for each maturity with different initial conditions $V_i(0)$
  (only the wiener processes are identically).

- note 2: caplet smile for 13 maturities with one set of parameters:
  - three parameters (β, λ, η) for the smile/skew
  - four parameters (a, b, c, d) for the term structure of at-the-money or fixed strike caplet
  - addtional optional 13 parmaters ($V_i(0)$ close to one) for perfect calibration of the term structure

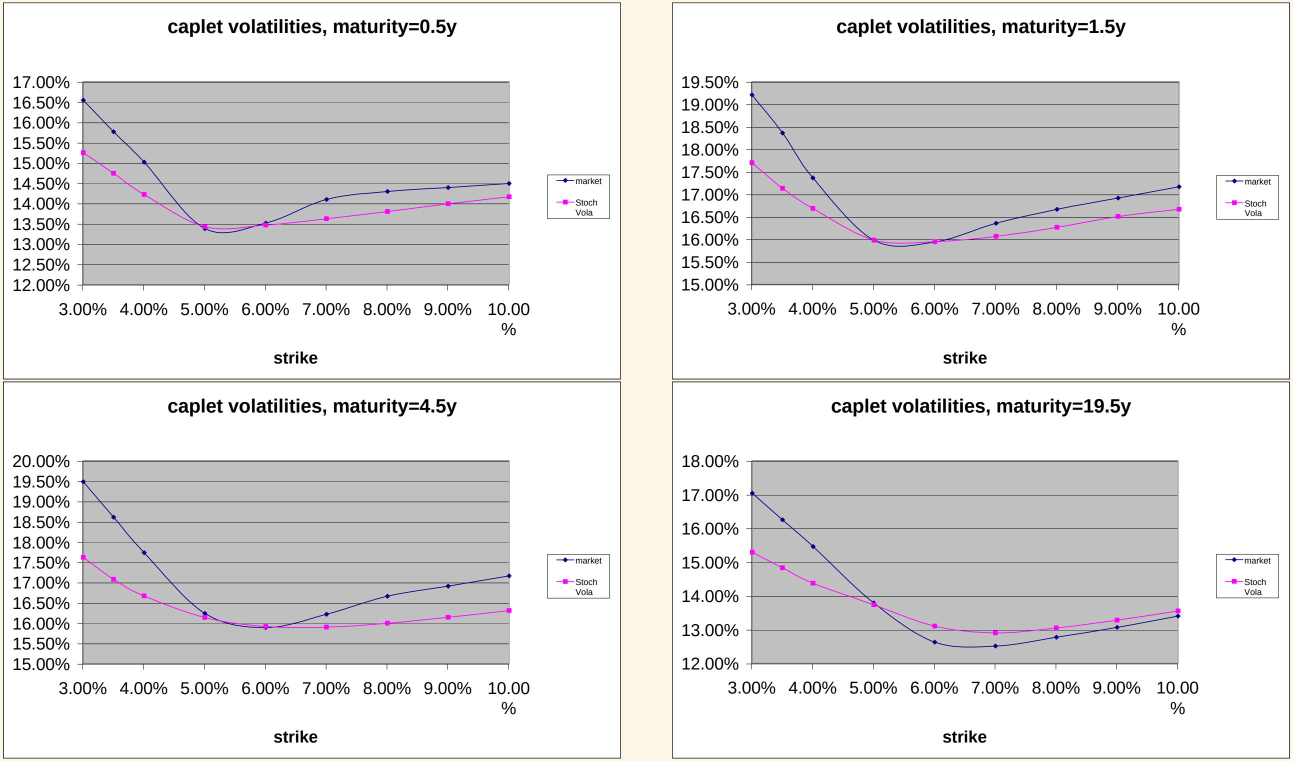


Figure 8: Calibration of the caption smile with one set of parameters. The initial values for the stochastic volatility function where chosen to fit the 5% strike values almost perfectly.

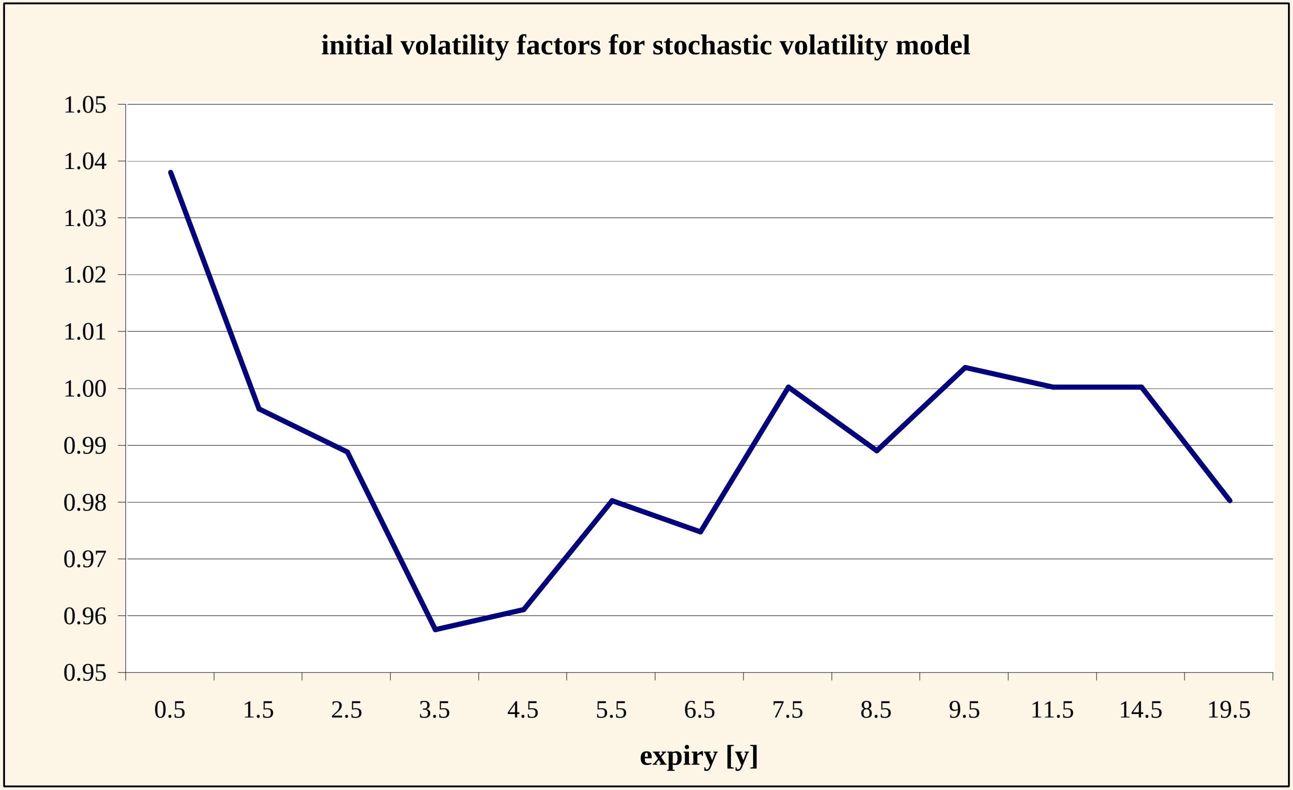


Figure 9: These initial values for the volatility process should be close to one. Deviations from one are needed for perfect fitting of the at-the-money or fixed strike caplet volatilities.

5. Conclusions

- calibration of the deterministic volatility Libor market model is well understood and works
- calibration of the stochastic volatility Libor market model is unsatisfying, but
  - not yet implemented with more than three additional parameters for the smile
  - using intitial non stationary variance values seems to be a good idea compared to having a non time homogenous volatility function (todo: backtesting!)
- todo: calibration to the swaption smile