Yield estimates for ABS-Transactions

Risk Management Breakfast

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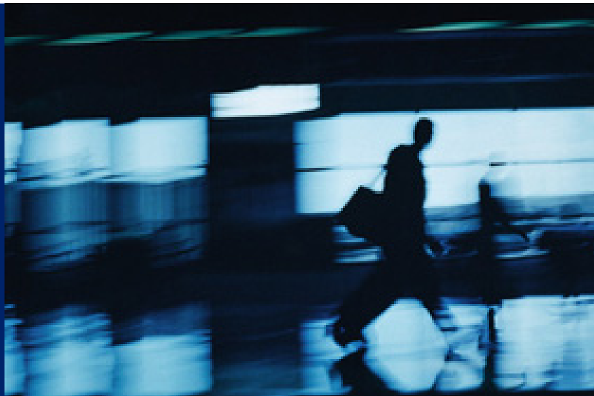


Agenda

- Overview Asset Backed Securitisation
  - Motivation
  - Composition of investor returns
- Yield estimates
  - Bad (but quite common) practice
  - Better practice - Factor-Model
  - Better practice - Fourier-Methods
- Future prospects
- Macroeconomic considerations - up-and-in barrier option modelling of embedded call options

Motivation

KfW expects a loss of 3.5 Billion Euro from IKB due to the US-ABS-Engagement

(SZ, 18.08.2007)

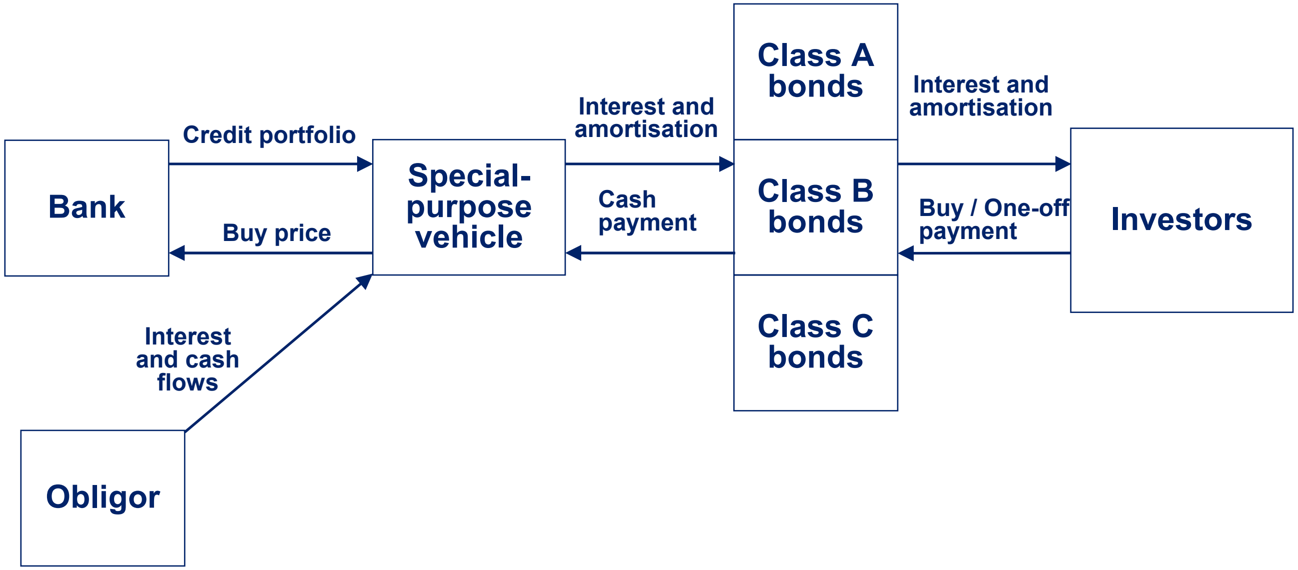
ABS stimulates profits

(FTD, 06.06.2007)

Savings institutions support Sachsen LB with 17.3 Billion Euro after ABS speculation

(Compliance Magazine, 20.08.2007)

Investor yields - dependent on which figures?



- Interest payments from the credit portfolio
- Administrative costs
- Cash flows
  - Probability of default (PD) of the credit portfolio
  - Loss given default (LGD) of the credit portfolio

Yield estimates - bad (but quite common) practice (1)

Class	Nominal [EUR]	Tranche thickness	Interest (float)
A	450,000.000	75%	4.0%
B	90,000.000	15%	6.0%
C	**60,000.000**	**10%**	**-**

Interest rate (attributed to the total nominal N): 3.9%

Fees: 600,000 EUR (0.1%)

Portfolio-Interest s: 8.5%

Portfolio-PD: 2.5%

Portfolio-LGD: 80%

Expected Loss L: 2% **?**

→ Expected yield of class C:

(8.5% - 3.9% - 0.1% - 2%) x (600,000.000 / 60,000.000) = 2.5% x 10 = **25%** **?**

Yield estimates - Bad (but quite common) practice (2)

Scenario: A prime interest rate increase of 1% occurs 6 months after setting up the transaction

Class	Nominal [EUR]	Tranche thickness	Interest (float)
A	450,000.000	75%	5.0%
B	90,000.000	15%	7.0%
C	60,000.000	10%	-

Interest rate (attributed to the total nominal N): 4.8%

Fees: 600,000 EUR	(0.1%)	Fees: 600,000 EUR (0.1%)	
Portfolio-Interest s:	8.5%	Portfolio-Interest s:	9.5%
Portfolio-PD:	2.5%	Portfolio-PD:	3.0%
Portfolio-LGD:	80%	Portfolio-LGD:	80%

→ Expected yield of class C:

(9.5% - 4.8% - 0.1% - 2.4%) x (600,000.000 / 60,000.000) = 2.2% x 10 = **22%**

Better practice - Loss distribution from 1-Factor-Model (1)

1-Factor-Model[1]**:**

Representation of the asset value Y_n of obligor n by the sum of a market component x common to all obligors of a given pool and an obligor-specific random component Z_n:

$$Y_n(t) = \sqrt{\rho} \cdot x(t) + \sqrt{1-\rho} \cdot Z_n(t)$$

- Market and random component are assumed to be normally distributed
- Z_n show no pair wise correlation and are not correlated with x
- Weighting of market and random component by asset correlation ρ

$\longrightarrow$ All Y_n are normally distributed and *independent for a given value of x*!

Default: time value of assets Y_n < time value of liabilities $\longrightarrow$ „Default threshold" α_n

$$\alpha_n = \Phi^{-1}(p_n(T))$$

with the probability of default p_n over time T and the inverse cumulative standard distribution Φ^{-1}

[1] See L. McGinty, R. Ahluwalia (2004): A Model for Base Correlation Calculation. JP Morgan London

Better practice - Loss distribution from 1-Factor-Model (2)

Calculation of the *average* default rate of loans in the portfolio over time T:

$\bar{p}(T) = 1 - e^{-cT}$ with the "clean spread" c: $c = \frac{s}{LGD}$

Approximation is acceptable for a homogeneous portfolio with a large number of obligors → Large Homogeneous Pool Model

From the 1-Factor-Model, the PD of the portfolio for market scenario x is thus:

$$p(x,T) = \Phi\left(\frac{\Phi^{-1}\left(\bar{p}(T)\right) - \sqrt{\rho} \cdot x}{\sqrt{1-\rho}}\right)$$

=> L(x): $L(x) = \min\left(\max\left(p(x,T) \cdot LGD, 0\right), \zeta\right)$

Better practice – Loss distribution from 1-Factor-Model (3)

Integration over all market scenarios results in the expected loss:

$$L = \int_{-\infty}^{\infty} \frac{d\Phi(x)}{dx} \cdot L(x) dx$$

Integration corresponds to the representative average of all possible market scenarios x, each weighted with the normally distributed probability of occurrence.

→ Expected yield of class C:

(8.5% - 3.9% - 0.1% - 3.14%) x (600,000.000 / 60,000.000) = 1.36% x 10 = **13.6%**

Standard deviation of L: 0.82%

→ Within just one standard deviation the yield varies between 5.4% und 21.8%; 1.7-times the standard deviation can already lead to losses

Where does the difference come from?

- Point estimation vs. distribution:
  - With "bad practice" methodologies the expected yield is estimated as a mean value through the use of averaged parameters
  - The described 1-Factor-Model gives a loss distribution and not a single point-like estimate.
- Consideration of the asset correlation in the 1-Factor-Model

However the described 1-Factor-Model uses

- an average probability of default ?
- a „global“ asset correlation, i.e. a correlation identical for all obligors ?

Better practice - Fourier-Methods (1)

Consider a portfolio of N correlated assets with nominals S_k and probabilities of default p_k where k = 1, 2, 3, …, N in the 1-Factor-Model:

$$p_k(x,T) = \Phi\left(\frac{\Phi^{-1}\left(p_k(T)\right) - \sqrt{\rho_k} \cdot x}{\sqrt{1-\rho_k}}\right)$$

The introduction of a „default indicator[1]" X_k leads to the portfolio loss rate:

$$PLR = \frac{S_1 X_1 + S_2 X_2 + \ldots + S_N X_N}{S_1 + S_2 + \ldots + S_N} = \sum_{k=1}^{N} s_k X_k \qquad \text{with} \quad s_k = S_k \Big/ \sum_{k=1}^{N} S_k$$

X_k are Bernoulli-distributed, $X_k = 1$ with p_k, and $X_k = 0$ with $1 - p_k$

[1] See A. Debuysscher und M. Szego (2003): The Fourier Transform Method - Overview. Moody's London

Better practice - Fourier-Methods (2)

Fourier transformation of PLR results in:

$$\hat{f}(t) = E\left[e^{-itPLR}\right] = E\left[e^{-it(s_1 X_1 + s_2 X_2 + \ldots + s_N X_N)}\right]$$

$$= E\left[\prod_{k=1}^{N} e^{-its_k X_k}\right] = \prod_{k=1}^{N} E\left[e^{-its_k X_k}\right]$$

X_k are independent random variables for a given market scenario x!

„Solving" for the expectation value with $X_k = 1 \wedge X_k = 0$ for a market scenario x:

$$\hat{f}(t) = \prod_{k=1}^{N} \left[p_k(x)\left(e^{-its_k} - 1\right) + 1\right]$$

Integration over the market scenarios:

$$\hat{f}(t) = \int_{-\infty}^{\infty} \prod_{k=1}^{N} \left[p_k(x) \cdot \left(e^{-its_k} - 1\right) + 1\right] \Phi(x) dx$$

Numerical solution via Gauss-Hermite-Integration

Better practice - Fourier-Methods (3)

Calculation of $\hat{f}(t)$ for different t gives the Fourier vector:

$$\begin{pmatrix} \hat{f}(t_1) \\ \hat{f}(t_2) \\ \hat{f}(t_3) \\ \dots \\ \hat{f}(t_l) \end{pmatrix} \xrightarrow{\text{Inverse Fourier transformation}} \begin{pmatrix} \varphi(a_1 = 0) \\ \varphi(a_2) \\ \varphi(a_3) \\ \dots \\ \varphi(a_l = 1) \end{pmatrix}$$

φ(a) corresponds to the probability distribution of the PLR

→ Expected yield of class C:

(8.5% - 3.9% - 0.1% - 3.48%) x (600,000.000 / 60,000.000) = 1.02% x 10 = **10.2%**

Standard deviation of L: 0.74%

→ Within only one standard deviation the yield varies between 2.8% and 17.6%; 1.4-times the standard deviation can already lead to losses.

Where does the difference come from?

- Consideration of the individual default probabilities and losses
- No global asset correlation

→ Fourier methods are more suited to small heterogeneous pools of securitised assets.

	"Flat"	LHP	Fourier
L	2%	3.14 ± 0.82%	3.48 ± 0.74%
Yield	25%	13.6 ± 8.2%	10.2 ± 7.4%

Future prospects

- Consideration of interest effects due to defaults in the portfolio
- Is asset correlation really independent from the market scenario?
- Introduction of a loss distribution function for different tranches.
- Consideration of macroeconomic factors in sub-senior tranche valuation models - motivated by current subprime crisis

Macroeconomic considerations (1)

- Consider ABS transactions with a subprime reference pool
  - Within the current subprime crisis most of the senior notes' OC and IC tests fail so investors in senior notes could make use of embedded "clean-up" call options
  - If those call options were executed in the current market situation standard valuation models will give a sub-senior tranche value close to zero
- Necessity of impairing sub-senior notes down to near-zero value?
  - Despite OC / IC tests failing a majority of call options embedded in senior notes have not been executed
  - Consider possible motives for this, focus especially on large transactions

→ With senior call options not being executed investors in FLPs or mezzanine tranches face the problem of finding an adequate fair value for their tranche exposures

Macroeconomic considerations (2)

- Assume that call options are not executed as option holders fear negative macroeconomic effects in case of execution in current market conditions
  - Above certain thresholds losses incurred in FLPs and mezzanine tranches may lead to bankruptcy of respective notes' holders
  - If large enough bankruptcies may backfire on option holders
- Bankruptcy threshold gives rise to a barrier for call option execution
  - Seen from current market conditions this motivates call option modelling via an up-and-in barrier option model
  - Barrier defines new optimal execution time of call option and hence results in new option value
  - From this new option value an adequate pricing for sub-senior ranches can be derived even in current market conditions

Macroeconomic considerations (3)

- Consider a sub-senior note holder within the up-and-in barrier option model
  - Default threshold of note holder is known
  - Therewith optimal call option execution time *as well as tranche losses upon option execution* can be determined
  - Discounting that future tranche value will give rise to current fair value of tranche held
- Presented model will give intrinsic tranche value - independent of current irrational market quotes - therewith potentially suited for e.g. accounting purposes

Macroeconomic up-and-in barrier modelling of embedded call options allows for a much more adequate tranche pricing than classical models - even in current "panic"-driven markets

Summary

- Careful consideration must be paid to the valuation model chosen - no "one size fits all" solution exists
- Wrong model might incur unexpected losses - seen from the investor's side
- Current subprime crisis motivates the incorporation of macroeconomic factors into tranche valuation models
- By giving the intrinsic tranche value those extended models are potentially suited for e.g. accounting purposes

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