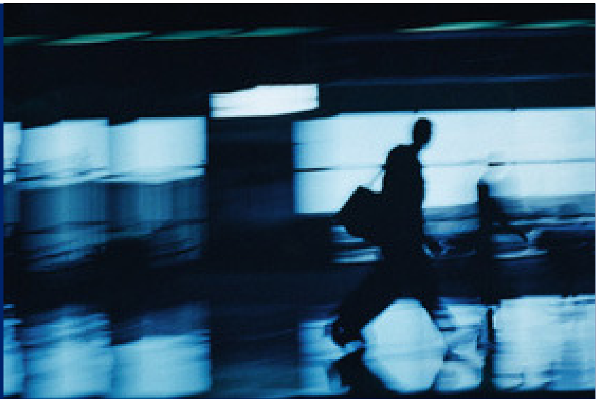
Merton Style Factor Model

Aspects of implementation & application

Capital Allocation 2007, Sep 17th 2007, London

Dr Georg Stapper



Agenda

- Motivation
- Factor Model and Correlations
- Efficient Monte Carlo Sampling Methods
- Stress Testing

Strategic target of risk and capital management

Target of the risk & capital management of a bank is to **assign adequate capital** to those transactions, which continuously provide **earnings exceeding capital costs**.
This way each transaction supports the increase of shareholders' value.

Detlef Bindert, Group Treasurer
Deutsche Bank AG, in Gabler 2004

Requirements

- portfolio model for group wide capital assessment
- risk sensitive capital allocation techniques down to transaction level
- group wide integration and allocation of capital for different risk types
- risk adjusted pricing tool
  (forward looking steering/backward looking measurement)
- manage Basel II capital point in time through the economic cycle
- further tools for Active Portfolio-Management
  (capital impact study, risk transfer pricing)

Basel II implementation standards

Economic Capital Allocation Requirements

Requirements for Credit Risk

EC contribution should scale with the credit “riskiness” of the transaction

- Transactions with lower credit quality should consume more capital
- Transactions with higher correlations/concentration risk should consume more capital

Fulfilled by **Coherent Risk Measure Expected Shortfall**
but not by Var/Covar allocation

(Artzner, Delbaen, Eber & Heath, 1997/99)

EC Allocation methods

Expected Shortfall:

- EC contribution of a transaction is the average loss of the transaction in the "extreme loss scenarios" of the portfolio:

  $ES(transaction) =$
  $E[L_{trans} \mid L_{portfolio} > quantile_{\alpha}] - E[L_{trans}]$

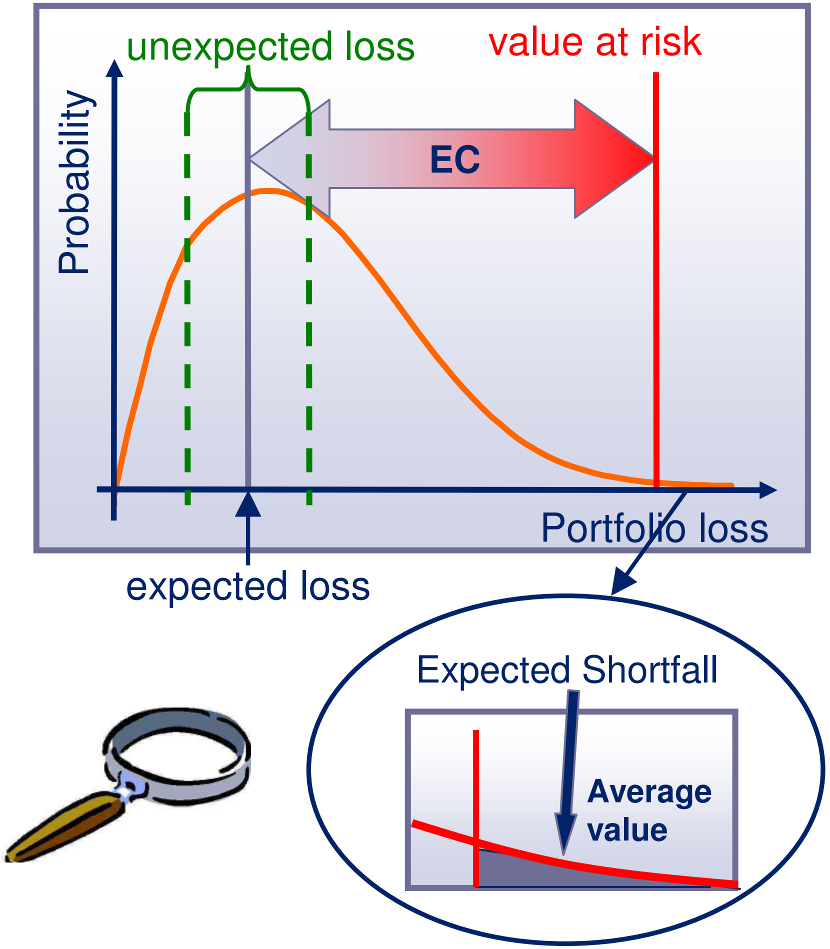
- distributes extreme losses

Unexpected Loss (stdev):

- Covariance Allocation:

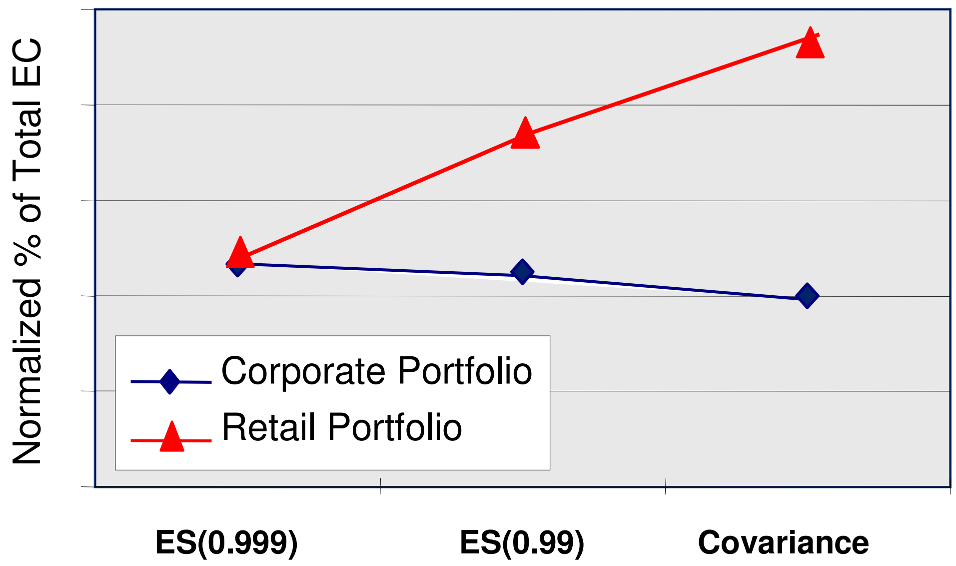
  $Cov[L_{trans}, L_{portfolio}] / Var[L_{portfolio}]$

- distributes loss volatility



EC Impact Study

typical financial institution, typical portfolios

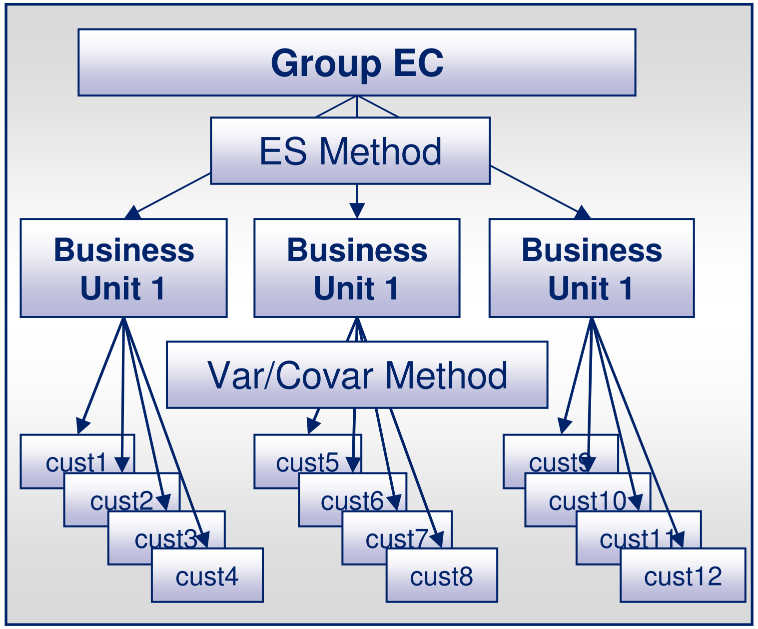


Concentration risk estimated by ES

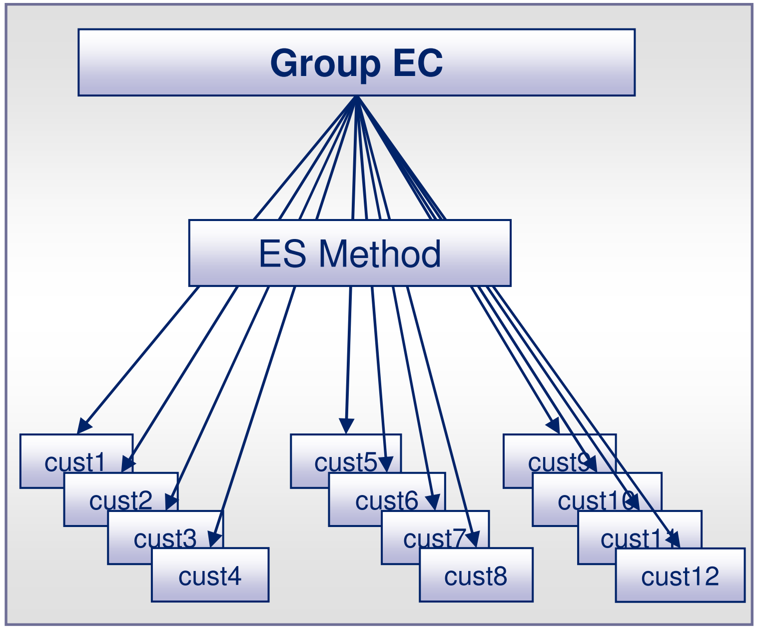
- EC increase in more risky trading portfolios (higher concentration risk)
- EC reduction in diversified retail banking
- Not just a technical issue: impact of management input – e.g. choice of threshold

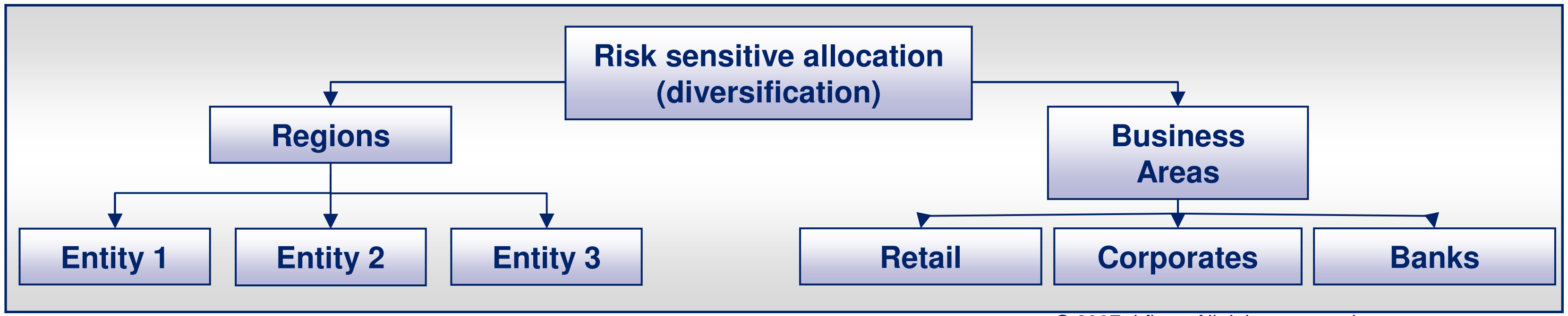
Goal: EC allocation down to transaction level

Mixture allocation technique



Pure Expected Shortfall allocation





Agenda

- Motivation
- Factor Model and Correlations
- Efficient Monte Carlo Sampling Methods
- Stress Scenarios

Estimation of Default Correlations

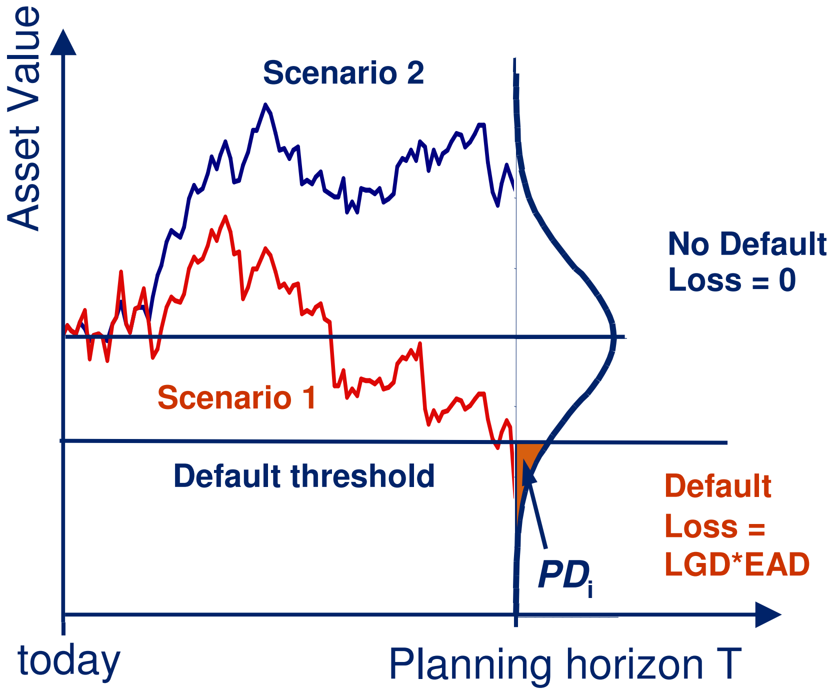
Directly from defaults or rating migrations

- Problem: too few joint default or migration events
- Really long history required, no relation to current economic cycle

Structural approach (Merton type model)

- Equity values widely available
- No linear relationship between equity and asset values
- Apart from credit risk several other factors influence the equity values which are not directly related to asset values

Merton, Default and Monte Carlo



Merton Model

- Default occurs when value A_i of firm's *i* assets falls below the value of its liabilities D_i
- Asset value modelled as Brownian Motion process (log returns normally distributed: $r \sim N(\mu, \Gamma)$)
- Value of liabilities fixed over time

Normalised Asset log Return Process

- Normalisation, discretisation → $\tilde{r}_i$ standard normally distributed, N(0,1)
- Value of liabilities calibrated such that

$$PD_i = \Pr\left[\tilde{r}_{i,T} < \tilde{D}_i\right] = \mathrm{N}\left[D_i\right]$$

Default Correlation: Factor Model

Factor Model: $$\tilde{r}_i = \underbrace{R_i \cdot \sum_{j=1}^{K} w_{ij} \cdot f_j}_{\text{systematic risk (country \& industry)}} + \underbrace{\sqrt{1-R_i^2} \cdot \varepsilon_i}_{\text{Specific (idiosyncratic) risk}}, \qquad \tilde{r}_i, f_j, \varepsilon_i \sim \mathrm{N}(0,1)$$

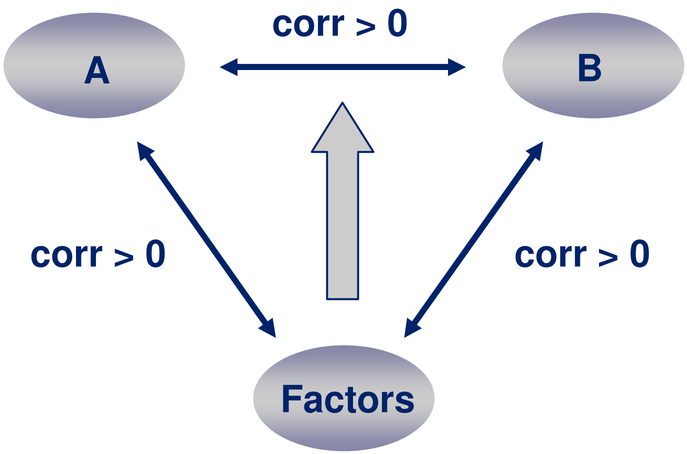
Decompose Firm's risk:

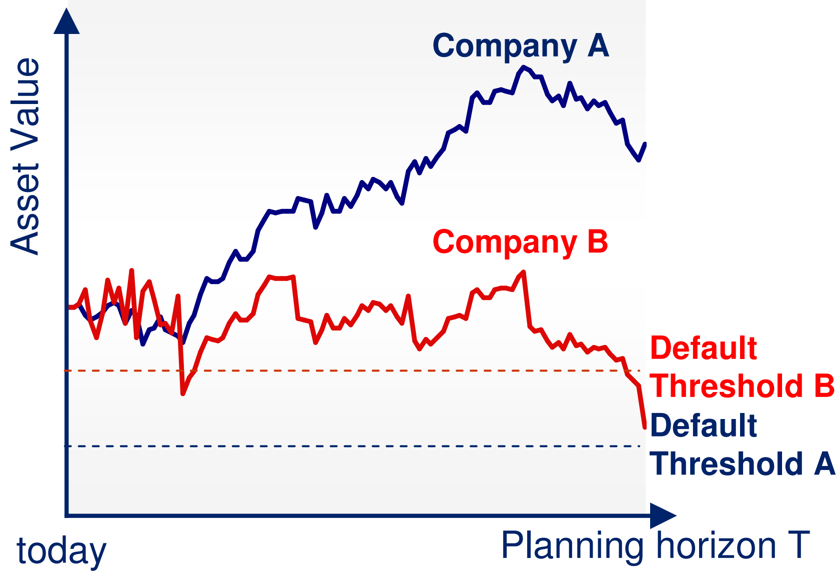
- Systematic Risk: Country & industry risk factors f_j. R quantifies to which extent the firms risk is explained by the factor model
- Specific Risk: independent part of firm value (ε_l), not correlated with creditworthiness of other counterparties
- Definition of Loss of a transaction: $$L_{\text{trans}} = LGD_{trans} \cdot EAD_{\text{trans}} \cdot \mathbf{1}_{(\tilde{r}_{i,T} < \tilde{D}_i)}$$
- Definition of Portfolio Loss: $$L_{\text{Portfolio}} = \sum_{i=1}^{N} LGD_i^{\text{trans}} \cdot EAD_i^{\text{trans}} \cdot \mathbf{1}_{(\tilde{r}_{i,T} < \tilde{D}_i)}$$

Normalised asset return Correlation ρ_{ij}: m Counterparties: m(m-1)/2 Correlations

Data Reduction in Factor Model: m ~ 25-100 factors

Factor Model Correlation: Example





assets (A) = 0.8 Germany + 0.2 US
+ 0.9 utility + 0.1 x finance
+ specific risk (A)

assets (B) = 0.6 US + 0.4 Japan
+ 0.3 utility + 0.6 x finance
+ 0.1 aerospace
+ specific risk (B)

Joint **D**efault **P**robability: JDP_{ij} :

$$JDP_{ij} := \Pr[\tilde{r}_{i,T} < \tilde{D}_i, \tilde{r}_{i,T} < \tilde{D}_j]$$

$$= N_2[N^{-1}(PD_i), N^{-1}(PD_j), \rho_{ij}]$$

Correlation of Asset return processes

d-fine

Default Correlations

Asset return correlations

joint probability distribution model (binary Gaussian copula) for asset log returns (σ_i, σ_j = 1):

$$\mathrm{JPD}_{ij} = \Pr(\tilde{r}_{i,T} < \tilde{D}_i, \tilde{r}_{j,T} < \tilde{D}_j) = \frac{1}{2\pi\sqrt{1-\rho_{ij}^2}} \int_{-\infty}^{\tilde{D}_i} dx \int_{-\infty}^{\tilde{D}_j} dy \exp\left\{ -\frac{1}{2(1-\rho_{ij}{}^2)} \left[\tilde{r}_{i,T} + \tilde{r}_{j,T} - 2\rho_{ij}\tilde{r}_{i,T}\tilde{r}_{j,T} \right] \right\}$$

default correlation (PD_i, PD_j Bernoulli distributed):

$$\hat{\rho}_{ij} = \frac{\mathrm{JPD}_{ij} - \mathrm{PD}_i\,\mathrm{PD}_j}{\sqrt{\mathrm{PD}_i(1-\mathrm{PD}_i)}\sqrt{\mathrm{PD}_j(1-\mathrm{PD}_j)}}$$

R^2 estimation for public and private companies

Asset log return process: $r = \beta \cdot X + \sigma \cdot \varepsilon$

Goodness of fit of regression between asset log return and composite factor *X* : $R^2 = \dfrac{\beta^2 \cdot Var(X)}{Var(r)}$

Normalize asset log return processs to standard normal:

$$\underbrace{Var(r)}_{\text{firm's total risk}} = \underbrace{\beta^2 \cdot Var(X)}_{\text{systematic risk}} + \underbrace{\sigma^2 \cdot Var(\varepsilon)}_{\text{specific risk}} = 1$$

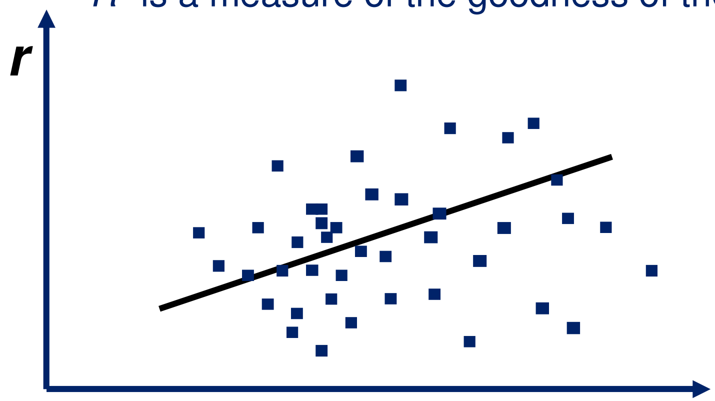
$$\Rightarrow \sigma = \sqrt{1-\beta^2 \cdot Var(X)} = \sqrt{1-R^2}$$

Approach 1: Take rough best practice values

Approach 2: Regression analysis

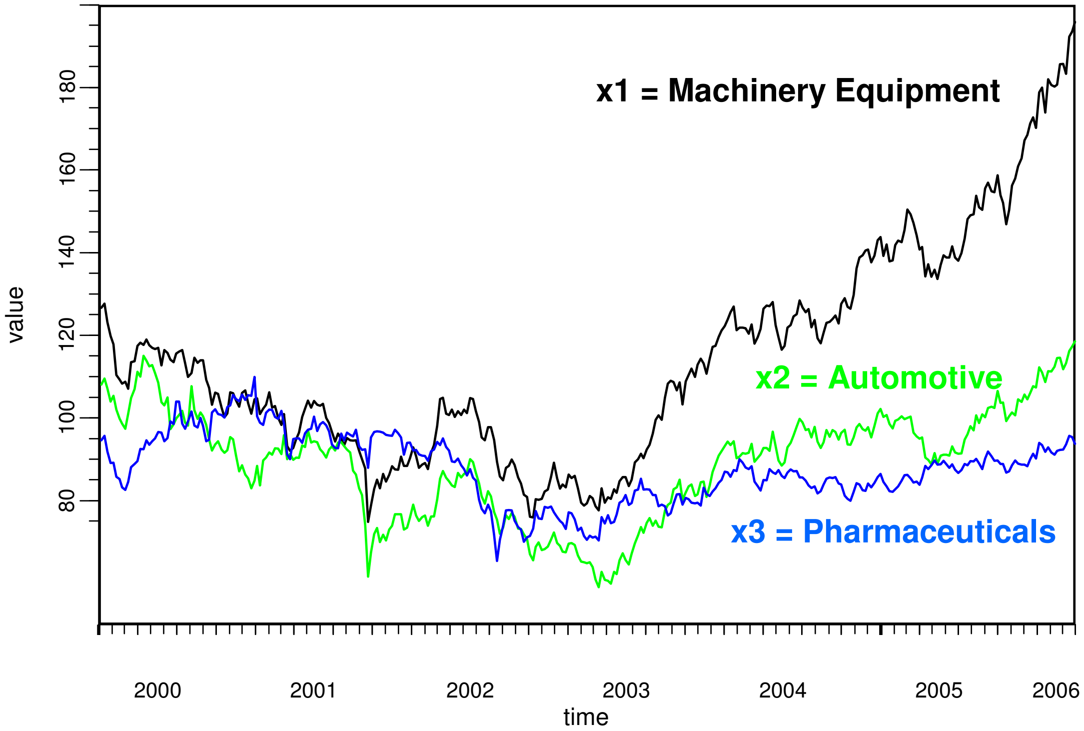
- Public firms
  - Regression between equity return value and factor model representation.
- Private firms
  - Regression between size and R^2 from above
  - Mapping to available public firms

R^2 is a measure of the goodness of the fit



Estimation of covariance matrix of a factor model

Illustrative example



Correlation Matrix

	x1	x2	x3
x1	1	0.76	0.43
x2		1	0.38
x3			1

Covariance Matrix (weekly)

	x1	x2	x3
x1	0.0007	0.0006	0.0003
x2	0.0006	0.0008	0.0003
x3	0.0003	0.0003	0.0005

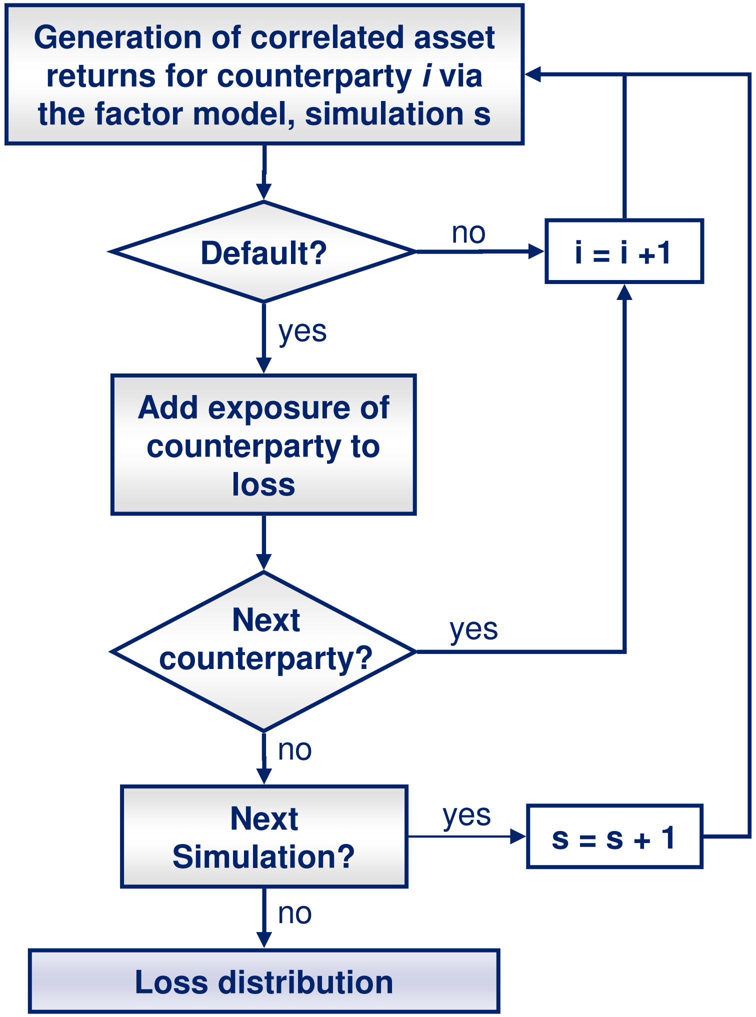
x 52 weeks

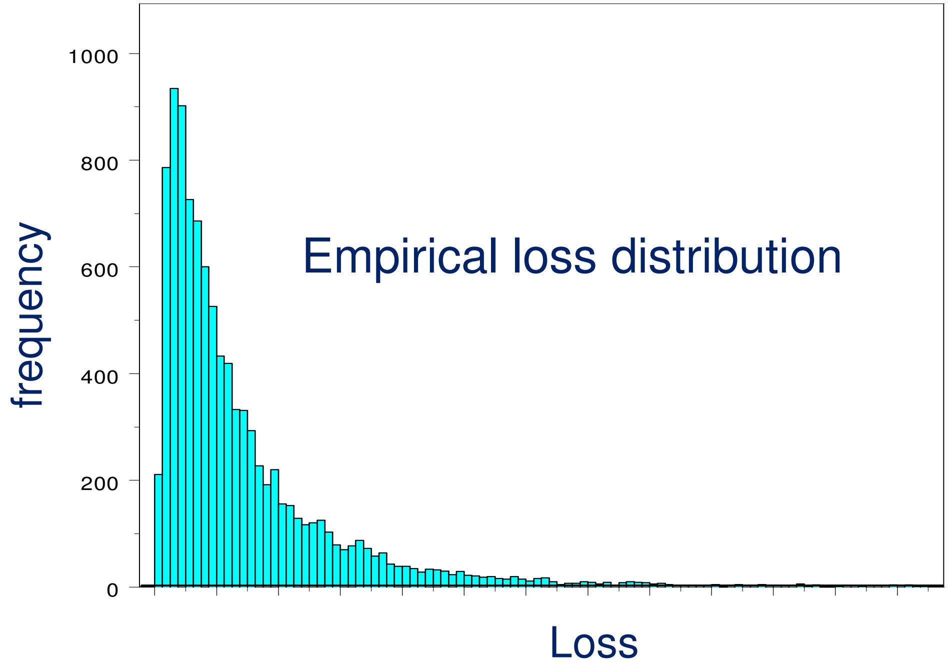
Covariance Matrix (yearly)

	x1	x2	x3
x1	0.0351	0.0293	0.0134
x2	0.0293	0.0427	0.0130
x3	0.0134	0.0130	0.0280

Estimate Covariance Matrix for the factors and scale with t=52 to get annualized values

MC Simulation of Loss Distribution for a credit portfolio





Agenda

- Motivation
- Factor Model and Correlations
- Efficient Monte Carlo Sampling Methods
- Stress Scenarios

Expected Shortfall

Expected shortfall of loss *L* at level $\alpha \in (0,1)$:

$$\mathrm{ES}_\alpha(L) = \mathrm{E}(L \mid L > \mathrm{VaR}_\alpha(L)) \approx (1-\alpha)^{-1}\,\mathrm{E}(L \cdot 1_{\{L>\mathrm{VaR}(L)\}})$$
$$= (1-\alpha)^{-1} \int L \cdot 1_{\{L>\mathrm{VaR}_\alpha(L)\}}\,\mathrm{dP}$$

Expected shortfall contribution of the i-th loan:

$$\mathrm{ESC}_\alpha(L_i) = \mathrm{E}(L_i \mid L > \mathrm{VaR}_\alpha(L)) \approx (1-\alpha)^{-1}\,\mathrm{E}(L_i \cdot 1_{\{L>\mathrm{VaR}(L)\}})$$

Example: α = 99.98% quantile, compute n =100000 MC samples s_i

s1 ≥ s2 ≥ … ≥ sn

$$\mathrm{ES}_\alpha(L) = (1-\alpha)^{-1}\,\mathrm{E}(L \cdot 1_{\{L>\mathrm{VaR}_\alpha(L)\}}) = (1-\alpha)^{-1} \int L \cdot 1_{[L<\mathrm{VaR}_\alpha(L)]}\,\mathrm{dP} = \sum_{i=1}^{20} s_i / 20$$

⇨ Only 20 samples fullfil the requirement for the total portfolio, but the contribution of the i-th loan is usually 0!

MC Acceleration Methods

- Parallel and multi-threading computing
  - Distributed memory
  - Message passing
- Variance reduction techniques
  - Antithetic variates
  - Importance sampling
  - Conditioning
  - others
- Quasi Monte Carlo

Kalkbrener, Lotter, Overbeck: “Sensible and efficient capital allocation for credit portfolios”, RISK (January 2004)

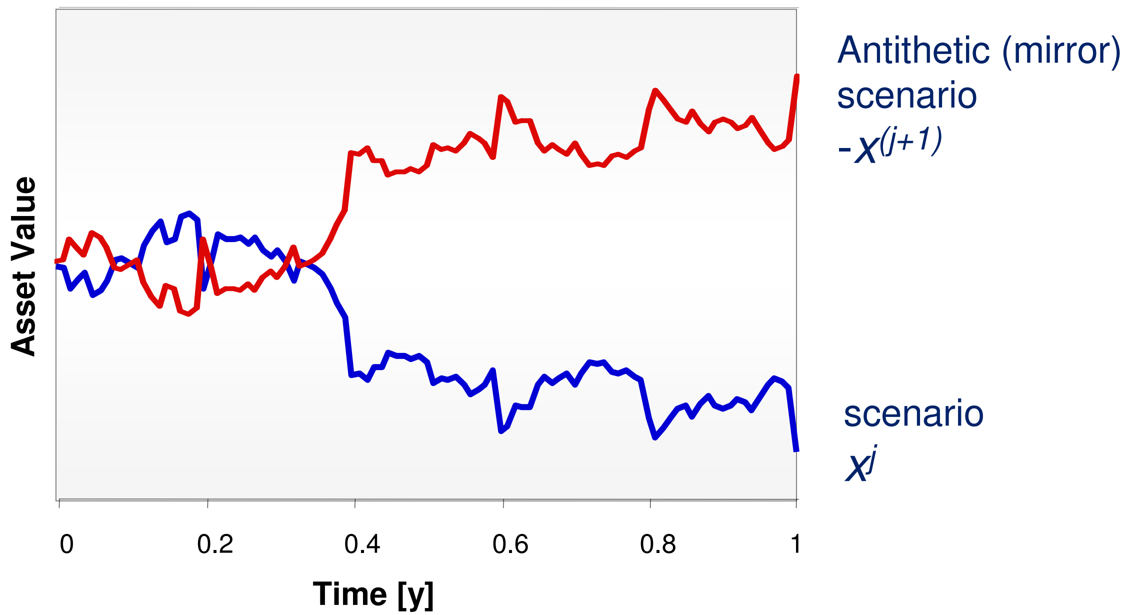
Egloff, Leippold, Jöhri, Dalbert: “Optimal importance sampling for credit portfolios with stochastic approximations”, Working paper, Zürcher Kantonalbank (2005)

Glasserman, Li: “Importance sampling for portfolio credit risk”, Working paper, Columbia Univ., New York (2003)

Glasserman, Kang, Shahabuddin: “Fast Simulation of Multifactor Credit Portfolio Risk”, (2007), Working paper

Antithetic variables

The principle of this simple technique is to add the factor scenario $-x^{(j+1)}$ as a new scenario of the simulation for every scenario x^j drawn from the simulation engine. We get one scenario for free !



Acceleration factor compared to crude Monte Carlo: ~ 2

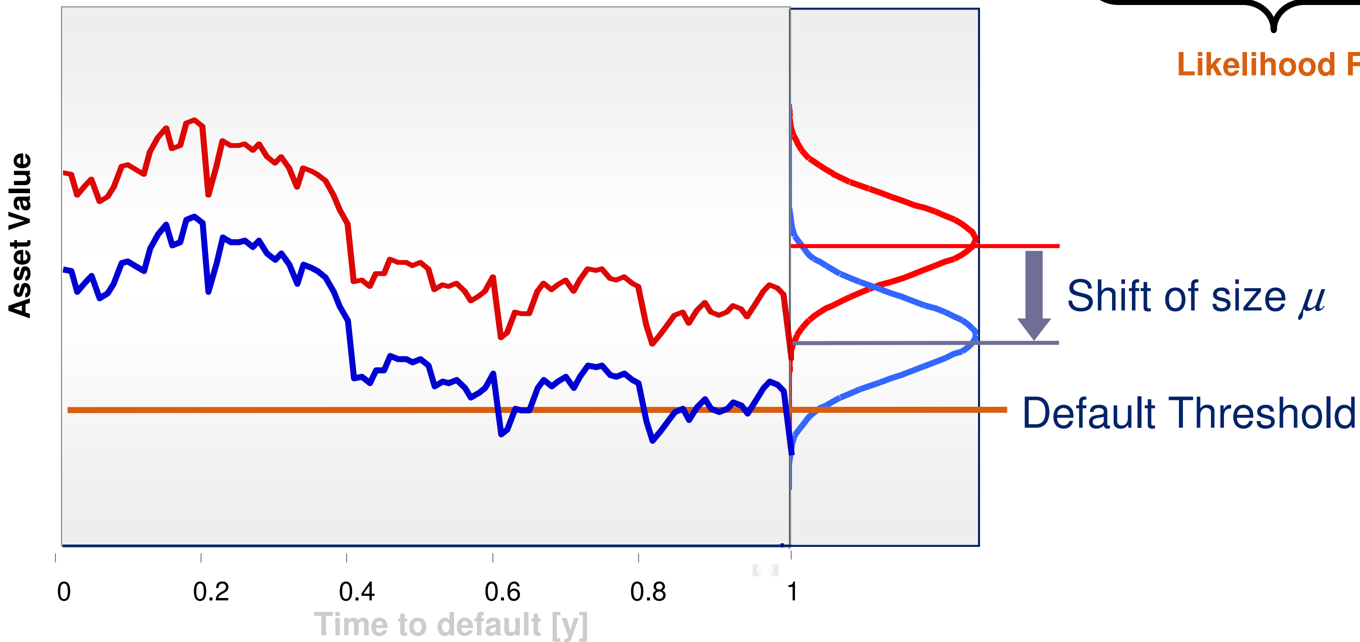
Importance Sampling: Shifting the systematic mean

Generate asset returns using N[-μ,σ] instead of N[0, σ] for the systematic factors. Amend Monte-Carlo estimate accordingly:

$$\mathrm{EL} \approx \frac{1}{m}\sum_{j=1}^{m} L^{(\text{scenario}\,j)}$$

Change of measure

$$\mathrm{EL} \approx \frac{1}{m}\sum_{j=1}^{m} \underbrace{\left\{ \frac{\mathrm{N}[0,\sigma]\left(\tilde{r}^{(\text{scenario}\ j)}\right)}{\mathrm{N}[\mu,\sigma]\left(\tilde{r}^{(\text{scenario}\ j)}\right)} \right\}}_{\text{Likelihood Ratio}} L^{(\text{scenario}\ j)}$$



Monte Carlo Acceleration combined

Crude Monte Carlo

- + Antithetic variable: factor 2
- + Importance sampling: factor 100 - 350
- + Quasi MC, conditioning and others factor ~ 40

- => Together (Ant.Var.* Imp.Sampl.*(Cond. and others))
- acceleration factor ~ 3000 - 20000
- Variance reduction: ~ $N^{1/2}$ = 55 - 140
  std error typically from 40% down to => 0.8% - 0.3%
- For stable ES allocation, a crude Monte Carlo run would need 750 mio simulations (run time ~ 15000 h for one EC run) instead of 100000 simulations (run time ~ 2h for one EC run)
- => ES is a stable allocation method!
- Depends on portfolio contributors and risk concentrations, i.e. for high risk concentration the acceleration factor gets bigger, for low concentration it gets lower

Agenda

- Motivation
- Factor Model and Correlations
- Efficient Monte Carlo Sampling Methods
- Stress Testing

Motivation and Business perspective of Stress Testing

Objectives

- Impact on credit reserve / expected loss and economic capital (EL, EC)
- Identification of concentration risk in sub portfolios (or other aggregation level)
- Identification of major capital consumers under stress

- Basel II: notes
  - Use of scenarios like economic or industry downturn, market risk events, liquidity shortage
  - Recession scenarios, not necessarily worst case scenarios
  - Banks should use their own data for rating migrations and integrate the insight of external ratings
  - Smaller deteriorations should also be considered

Stress testing framework

Rare but reasonable scenarios (e.g. 1 in 10 year stress event)

- All EC relevant risk types must enter into the regular stress testing framework
- Macroeconomic, business related and quantitative aspects to be considered with respect to risk profile implications
- Change of risk profile and its impact on capital requirements (incl. regulatory capital) should be analyzed
- Regular reporting to senior management and the business to act accordingly is the focus
- Stress testing report for group and sub-portfolios of special risk distinction are required on a daily, weekly and monthly basis

Common Practice & typical Problems

- Observed singular stress event
  - "Worst annual internal experience"
  - Broad default rate movements
  - Equity market crash
  - Widening of credit spreads
- Arbitrary shocks
  - e.g. impose a two-grade downward migration across the portfolio.
- Stress key parameters
  - e.g. increase all PD, LGD, EAD or correlations by 10%, 25%, 50% or 100%.
- Relevant data often not incorporated
  - e.g. internal transition matrices, internal economic projections, historical parameter behaviour
  - correlations between LGD and PD
  - volatilities of LGD, EAD
- Missing
  - Portfolio Dynamics
  - Economic Dynamics; multi sector correlations
  - Scenario probabilities

Example: Stress Transition Matrix

Historical event: e.g. Changes in transition matrices from 2000 to 2002

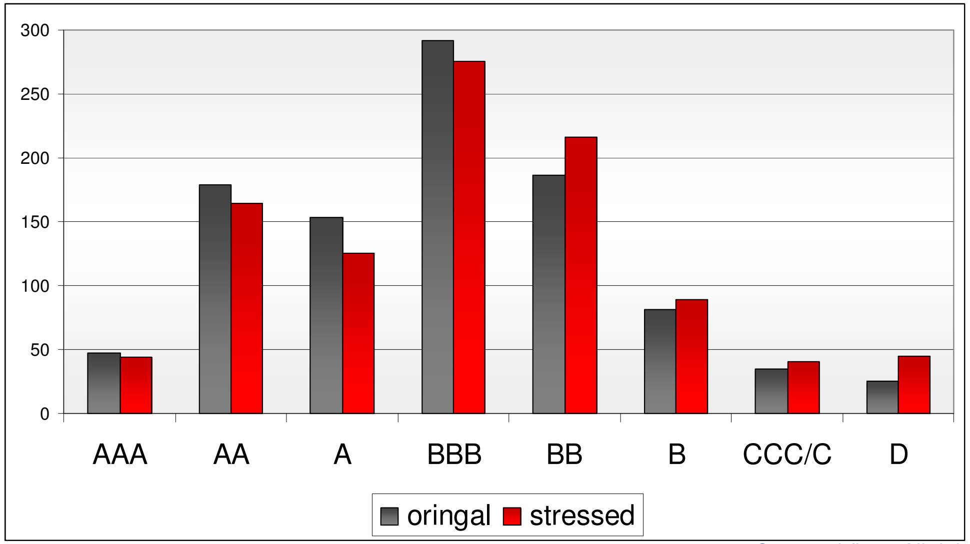
Original transition matrix

1 year	AAA	AA	A	BBB	BB	B	CCC	D
AAA	68.19	23.04	5.20	1.43	1.44	0.60	0.07	0.03
AA	6.29	72.00	13.13	6.60	1.27	0.30	0.32	0.09
A	1.31	22.85	42.70	25.72	5.83	0.71	0.30	0.58
BBB	0.27	4.25	13.17	62.06	18.25	0.67	0.41	0.92
BB	0.02	0.53	3.12	21.89	59.06	12.03	1.49	1.86
B	0.10	0.32	0.97	9.21	26.55	50.91	2.89	9.05
CCC	0.10	0.19	1.10	2.02	4.99	16.90	55.06	19.65
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

stressed transition matrix

1 year	AAA	AA	A	BBB	BB	B	CCC	D
AAA	62.00	27.40	5.74	2.68	1.44	0.60	0.07	0.07
AA	6.29	63.27	7.07	12.65	7.44	2.24	0.52	0.52
A	1.23	21.24	37.46	29.47	7.78	1.37	0.36	1.09
BBB	0.19	4.17	10.64	51.36	26.07	2.12	1.71	3.74
BB	0.02	0.30	3.00	21.60	54.80	13.54	2.58	4.16
B	0.10	0.32	0.66	8.72	26.35	47.57	5.61	10.68
CCC	0.10	0.19	0.52	2.02	3.83	16.88	49.27	27.20
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Expected Rating distribution



Constraints on sampling space

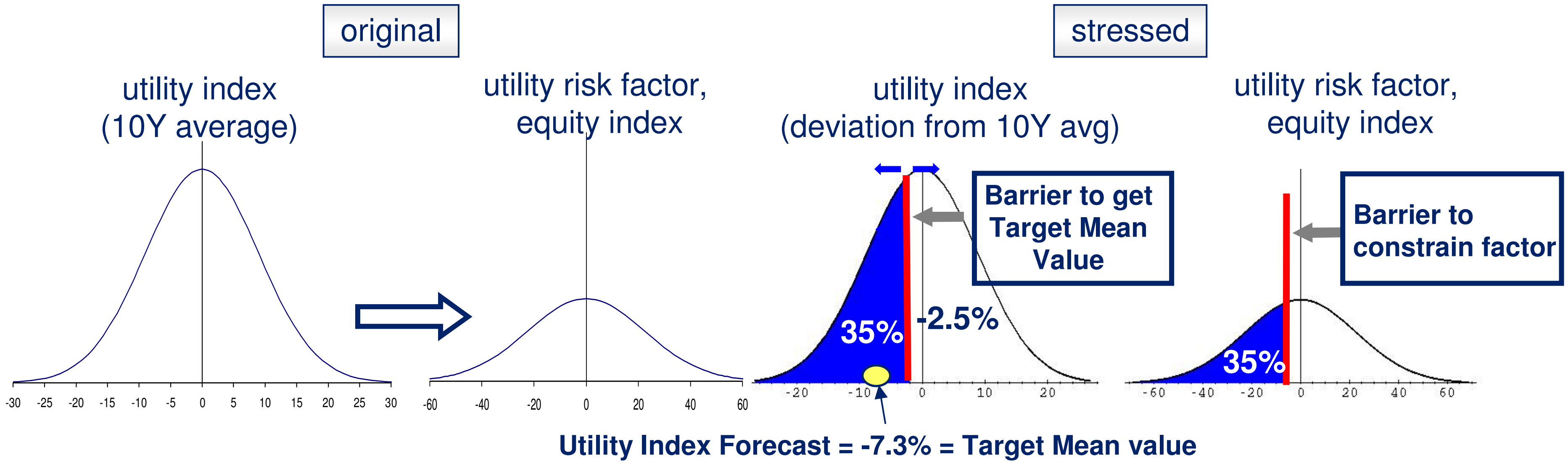
- Specify economic stress scenario determined by economists
  Example: decline of 20% in the utility index.
- Translate scenario into stress of systematic factor (restriction of factor space)
- Concentrate on a small number of relevant systematic risk factors
  (preserve probability of scenario occurrence)
- Other factors are impacted through correlations to the stressed factors.
- Restrict Monte Carlo sample according to the stress scenario

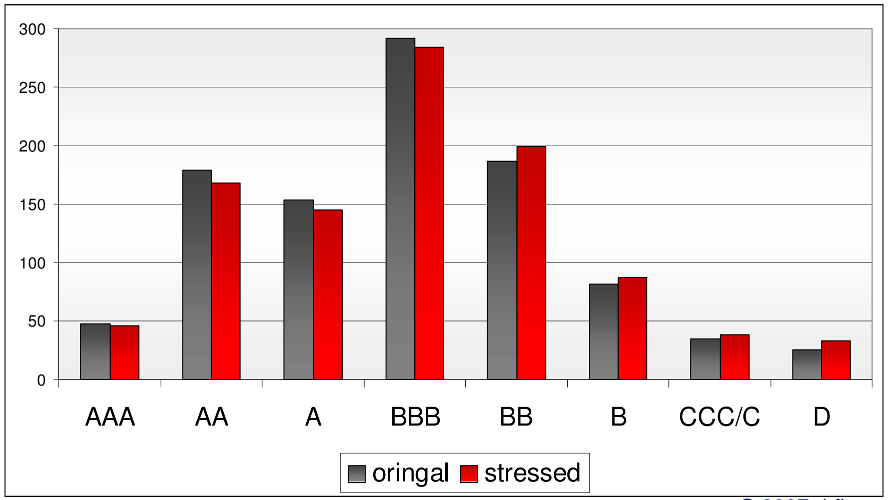
  ⇨ consistent set of stressed PDs for all loans in the portfolio,
  change in PD depends on the loan's correlation to the stressed factors

  ⇨ estimate for the probability of the simulated scenario (probability of occurrence)
- Determine impact of stress scenario by calculating expected loss conditioned on the scenario and other statistics of the portfolio.
  e.g.: $PD_{i,stress} = EL_{i,stress} / LGD / EAD$

Bonti, Kalkbrener, Lotz and Stahl: Credit Risk Concentrations under Stress, working paper, 2005

Link between Macroeconomic Outlook and Credit EC





Lunch!

Your Contact to d-fine:

Dr. Georg Stapper
Director

mobile +44-790 - 825 1912
phone +44-207 - 776 1004
georg.stapper@d-fine.de

Dr Christian Oehler
Manager

mobile +49-151 - 148 193 04
phone +49-69 - 907 373 04
Christian.oehler@d-fine.de

Dr. Bernd Appasamy
Managing Director

mobile +49-151 - 148 193 06
phone +49-69 - 907 373 06
bernd.appasamy@d-fine.de

d-fine Ltd

28 King Street

London

EC2V 8EH

www.d-fine.co.uk